Optimizing the True Market Portfolio*

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Abstract

This paper updates the global market portfolio per 2020, through revising already identified market portfolio asset classes, adding previously excluded asset classes, and studying the asset classes in further detail. Focus is on alternative and private market asset classes, which have been excluded from previous studies to a great extent while research shows the potential benefits of including these assets and investment trends are moving towards more private market investments, crypto currencies, and collectables. The results demonstrate an enhanced risk-adjusted return when including mentioned asset classes in portfolio construction, the findings further demonstrate certain reoccurring optimal asset classes. The optimal portfolios are on average robust, according to performed bootstrapping.

Key words: *global market portfolio, portfolio optimization, diversification, private markets, alternative investments.*
JEL classification*: E44, G11, G12, G20, N20*

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1. Introduction

This study contributes to current research through creating a current and more reliable proxy for the market portfolio in further detail and with a greater coverage relative previous studies. Focus is on private and alternative asset classes which has emerged in the last decade and therefore been relatively excluded from this type of research. This research benefits investor awareness, diversification possibilities, portfolio construction and cost of equity estimates. Further, no previous study regarding the market portfolio has aimed at identifying the optimal portfolio allocation, thus, the portfolio optimization in this paper will provide further insights in optimal portfolio construction and possible portfolio benefits of including private asset classes in a portfolio.

Documentation of the market portfolio has been conducted by a few scholars since the late 1900s (see Ibbotson and Fall, 1979; Ibbotson and Siegel, 1983; Ibbotson et al., 1985; Louis and Roncalli, 2012; Doeswijk et al., 2014, 2020; Gadzinski et al., 2018). The period, length, markets, and asset classes studied vary to a great extent across the papers. However, they all tend to exclude various asset classes, especially alternative and private markets, such as private infrastructure, certain commodities, certain fixed incomes such as catastrophe (CAT) bonds and collectables. The studies are also not very detailed as for the types of private and public equity, nor previously mentioned asset classes. However, scholars such as Louis and Roncalli (2012) and Ibbotson et al. (1985) highlight the gravity of a relevant and updated market portfolio. They demonstrate that previous return documentation for assets classes and the market portfolio have shown to be of importance for informational reasons (for example see Robeco (2021) use the market portfolio by Doeswijk et al. (2014) for information reasons in a monthly outlook), investor awareness, diversification, portfolio construction and cost of equity estimates (see the paper of Kamara and Young (2018) where the market portfolio of Doeswijk et al. (2014) is used for cost of

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equityestimates). Moreover, the market portfolio is constantly changing as investment trends are changing; McKinsey & Company (2020) demonstrates that the total assets under management (AuM) in private markets has reached new record levels with significant growth[1] in recent years; Knight Frank (2019) demonstrate a growth in collectable investments; and the market capitalization of cryptocurrencies has had an immense growth and more than doubled in the first quarter of 2021, from one to two trillion USD, with Bitcoin reaching one trillion USD (Bloomberg, 2021). Furthermore, Chambers et al. (2018) find that adding low correlation asset classes in portfolio construction will lead to diversification benefits[2]. Research also show the benefits of including private and alternative asset classes in portfolio construction as these demonstrate low correlation to traditional assets, e.g. stocks and bonds (Leitner et al., 2007; Chambers et al., 2018; McKinsey & Company ,2019; McKinsey & Company ,2020; BlackRock, 2020). Nevertheless, as previously mentioned, papers have excluded these asset classes to a great extent in the construction of the market portfolio which yields an incomplete market portfolio[3].

Documenting the global market portfolio will therefore provide a substantial contribution to current literature. As previously stated, existing papers on the topic have identified the market portfolio at a relatively broad level, generally not including alternative and private asset classes. However, reports (see McKinsey & Company, 2020; BlackRock, 2020) show the potential benefits of these investments along with a high growth of invested funds in these assets. This study will therefore provide more insights in the market portfolio, including previously excluded asset classes such as private infrastructure, hedge funds, cryptocurrencies[4], all commodities, all private real

[1] Total assets under management (AuM) in private markets reached a new record in 2019 at 6.5 trillion dollars, and between 2010 and 2020 AuM in private markets experienced a growth of 170 percent, while global public market AuM grew by approximately 100 percent (McKinsey & Company, 2020).
[2] Chambers et al. (2018) find that increasing a portfolio from including one to four asset classes reduces the total risk by 26 to 50 percent, if correlation coefficients are equal to or less than 0.4.
[3] The inadequate coverage of asset classes in previous research is likely due to computational problems of identifying investable asset classes along with relevant proxies for these. This is especially difficult regarding private markets since lack of transparency and reporting will result in deficient data availability. Computational problems along with data-availability provide an explanation for the non-inclusion of private market asset classes.
[4] For the purpose of studying the global invested landscape and considering the market capitalization of cryptocurrencies and the growing institutional demand, it is of interest to include the investment in the construction of the global market portfolio as an asset class. Further, more research indicating the possible benefits of

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estate including residential, commercial, timberland and agricultural as well as more fixed income assets. This study will also break down the asset classes in further sub-asset classes (see Appendix I). This paper will therefore document the global market portfolio in more detail than previous papers (see Ibbotson and Fall, 1979; Ibbotson and Siegel, 1983; Ibbotson et al., 1985; Louis and Roncalli, 2012; Doeswijk et al., 2014, 2020; Gadzinski et al., 2018), considering both broad asset classes (e.g. private and public equity, bonds, real estate, infrastructure etc) and the sub-asset classes (e.g. on geographical level or different types of investments). Another reason why this study contribute to current literature is the continuously changing investment trends; some investments were not possible to invest in when the first studies of the market portfolio were conducted, nor had it reached its current growth in invested funds or popularity[5]. The Covid-19 pandemic has also had an immense effect on the global invested landscape and some investment trends (see UNCTAD, 2020a, 2020b). Nevertheless, there is no study investigating the market portfolio since the pandemics outbreak. It is therefore relevant to update and document the global market portfolio, including previously excluded asset classes at a more recent date; to enrich the knowledge about global investments, focusing on private and alternative markets, and these asset classes effect in portfolio construction.

This paper will further use the identified market portfolio in Markowitz optimization[6] to find the optimal weights of the broad asset class and their sub-asset classes. This paper addresses three research questions; (1) what is the global market portfolio, (2) what is the optimal risky portfolio allocation of the identified market portfolio and (3) is there any over- or under-

cryptocurrencies in portfolio optimization, why it is of interest to also include the asset class in the portfolio optimization. The same applies for collectables.

[5] Cryptocurrencies was not possible investments when the studies by Ibbotson & Siegel (1983) and Ibbotson & Fall (1979) were conducted. Also, the digital currency has experienced its emergence after studies by, for example, Doeswijk et al. (2014) and Gadzinski et al. (2018) were conducted. When studying asset classes several years' worth of data is usually preferred as well. Another example is the growth of private and alternative asset classes in later years (see Knight Frank, 2019; McKinsey & Company, 2020; BlackRock 2020).

[6] The Markowitz optimization is performed under different constraints.

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performance of the optimal risky portfolio relative the identified market portfolio and a classical 60/40 stock-bond portfolio?

The remainder of this paper is organized as follows. Section 2 outlines the data and its collection along with applied methodology. Thereafter, the results will be presented in section 3, followed by a discussion in section 4 and lastly a conclusion in section 5. Moreover, the Appendices include a more detailed overview of the data, methodology and results.

2. Materials and method

2.1. Materials

This study is primarily based on 39 indices for the sub-asset classes of public equity, private equity, direct lending, private real estate, infrastructure, fixed income, commodities, collectables, cryptocurrencies, and hedge funds (See Table A1 in Appendix I for a full overview of the indices). These indices have been accessed in various ways depending on availability; FactSet, Thomas Reuters Eikon, Bloomberg, company websites and e-mail contact with companies providing index data. Total return data in USD was retrieved from the indices first value date[7] to December 2020. The total return data is retrieved on monthly or quarterly basis upon availability since certain indices of illiquid assets only record data on quarterly basis (See Table A1 in Appendix I). Quarterly data have been exponentially smoothed to be comparable to monthly data in the analysis. Further, the index market capitalization in USD was retrieved either directly for the index or from reports or other estimates (See Table A2 in Appendix I for market capitalization estimates). For calculations using the risk-free rate, the 10-year US Treasury rate is used, which corresponded to 0.93 percent on 2020-12-31 (U.S. Department of Treasury, 2021).

[7] The first value date is the date which the index first records a value. For some indices this is before the launch date and the base date, why it is important to make a distinction.

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Like previous studies (see Doeswijk et al., 2014, 2020; Gadzinski et al., 2018) indices are used as proxies, since constructed in order to reflect a certain market or asset objectively. Indices were chosen upon availability and relevance; after identifying possible indices for a certain asset class, the broadest index was chosen in terms of underlying market capitalization and number of constituents based on. In some cases, a preferred index could not be used due to data availability. See Appendix III (online Appendix) for a full overview of data collection and information of chosen proxies for return data, and Appendix IIII for a full overview of data collection and information regarding proxies and estimates for assets market capitalization.

2.2. Estimation errors

Important to note is that the findings of this paper, along with similar studies, are subject to estimation errors due to several factors. Firstly, when identifying the global market portfolio proxies for asset classes and sub-asset classes return series and market capitalization are used. Since the proxies cannot perfectly reflect an entire asset class the true return series and market capitalizations are likely to differ which causes a problem of representativeness. Appendix III further explains the proxies used for return series, their coverage, and compare these with other possible proxies. Appendix IIII explains the used estimations for market capitalization, and compare these with other estimates found, when applicable. Moreover, using historical return data as proxy for future returns is also subject to estimation errors since the actual returns are likely to differ. Therefore, a robustness test is carried out on the data through bootstrapping with resample (for further detail see 4.5 Robustness test). Secondly, estimation errors are more likely in regard to private market data and estimates, as private market assets are not as liquid and therefore difficult to make estimates for. Five of the proxies have return data on quarterly level (see Table A1 in Appendix I), these return series have thus been exponentially smoothened to be made comparable to other proxies on monthly level. Nevertheless, the standard deviation, which is a cornerstone in the Markowitz optimization, is likely to be affected by the quarterly data. For example, it is likely

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that the quarter recorded does not capture the swings and hence lower the standard deviation. The quarter recorded can hence indicate less or more volatility than the months not recorded, while less is more likely. Thirdly, the data is subject to survivorship bias in the cases that the indices only cover constituents that have "survived", e.g. not become bankrupt. Given that the indices cover constituents without (or limited) backfilling, this eliminates survivorship bias, as the constituents are included according to strict index rules. Reporting bias is another potential estimation problem, for which especially hedge funds are subject. This bias arises as only well-performing funds decide to disclose their performance, while worse-performing decide not to. This leaves data mainly based on the hedge funds performing good, biasing the estimates upwards. This is not the case for public markets, as companies are required to report. However, the risk can also arise for private markets, as reporting is not required as for public markets. Lastly, in the analysis the optimal portfolios are created as per 2020, and not rebalanced on annual basis, while the optimal weights are likely to change on a regular basis along with changes in standard deviation and expected return for the different assets.

The indices should nevertheless provide a good overview of the global investable market, complementing previous studies (see Ibbotson and Fall, 1979; Ibbotson and Siegel, 1983; Ibbotson et al., 1985; Louis and Roncalli, 2012; Doeswijk et al., 2014, 2020; Gadzinski et al., 2018) and reports (see McKinsey & Company, 2019; BlackRock, 2020). In particular, this paper includes more asset classes, asset classes in further detail (i.e. sub-asset classes) and in some cases broader proxies' relative previous studies. This should hence allow for meaningful results, especially in conjunction with the robustness test.

2.3. Methodology

The market portfolio

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The global market portfolio is constructed based on identified asset classes, and their market capitalizations for December 2020. Further, a return index series for the identified asset classes and the market portfolio is constructed, capped on the asset's weights in the market portfolio. Important to note is that index constituents may have changed over the period studied, as constituents are added or dropped. The net change of this is assumed to be neglectable for this research.

Portfolio construction

In this paper five optimal portfolios are created and studied: an equal weighted portfolio[8] and four optimal risky portfolios with different constraints. These portfolios include all identified asset classes. Thereafter, five benchmark portfolios are created based on the 60/40 stock-bond portfolio investment strategy[9].

The portfolio construction (equal weighting, risky portfolio optimization and 60/40 portfolios) is carried out based on return data for two time periods, 2014-2020, which includes all identified asset classes and sub-asset classes (39 indices), and thereafter 2003-2020, including all sub-asset classes with available data (30 indices). The periods are chosen upon data availability; the first possible year including all asset classes is 2014; and the second period is chosen upon availability, as any earlier year would lead to a loss of several indices. An alternative would have been to choose 2007-2009 as a starting period which would include more indices but

[8] Martellini (2008) finds that the optimal risky portfolio outperforms a market capitalization weighted portfolio, considering risk-adjusted returns. Similarly, there is research that shows that an equal weighted portfolio outperforms market capitalization- value- or price-weighted portfolios (see Plyakha et al., 2012; Bolognesi et al. 2013; Malladi & Fabozzi, 2017; Brown, n.d.). For example, DeMiguel et al. (2006) finds that optimal portfolios do not consistently beat equal weighted portfolios in terms of turnover, Sharpe ratio, and certainty-equivalent return. In a later study, DeMiguel et al. (2009) also demonstrates that out of sample equal weightings also outperform mean-variance weightings. Taljaard and Maré (2021) find that an equal weighted portfolio outperforms the market capitalized portfolio in the long-term, while in the short-term there are periods of significant underperformance.

[9] The 60/40 stock-bond investment strategy is commonly adopted amongst financial institutions such as pension funds and other investors for decades as the idea is to provide growth via the 60 percent weight to equity and stability through the 40 percent weight to bonds (Ambachtsheer, 1987; Bernsteinm, n.d.; Kaya, 2019; Kephart, 2021; McCormick, 2021; Toschi, 2021; Van Der Zwan, 2021). The investment rule have seen more criticism in recent years and been called for an update (Kaya, 2019; McCormick, 2021; Toschi, 2021).

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simultaneously drop four to six years of observations, including the global financial crisis. Also, when including private asset classes, the period a portfolio is held tends to be longer due to liquidity. Therefore, 2003 as a start year for the second period providing the best coverage while the longest time frame.

Moreover, the optimization is conducted in four steps, yielding four optimal risky portfolios for each period (2014-2020 and 2003-2020). Firstly, the optimization is carried out solely in accordance with above methodology to retrieve the optimal portfolio weights of the different asset classes and sub-asset classes. In this optimization the only constraint is that sum of the weights needs to equal 100 percent. Thereafter, the mean-variance portfolio optimization is conducted with constraints set to the range of the weights, for the sub-asset class. The second optimization will therefore follow the constraint of a minimum investment of one percent in each sub-asset class, which will yield further diversification. The third optimization adds an additional constraint of a maximum weight of 20 percent to a single sub-asset class, which will reduce risk associated to individual sub-asset classes and thus yield further diversification. This is important since the portfolio, even if it yields a high Sharpe ratio, is exposed to large risks with less diversification. Lastly, the weights of the portfolio asset classes will be capped based on their true weights in the identified global market portfolio (see Table A3 in Appendix I).

To benchmark the five optimal portfolios, including all asset classes, five 60/40 stock-bond portfolios are constructed. These portfolios hence only include public equity and investment graded fixed income[10]. The first portfolio is equal weighted, where equal weights are given within the asset classes so that the sum is 60 and 40 percent for public equity and fixed income, respectively. The three subsequent portfolios are run with Markowitz optimization, all with the constraint that the sum of the public equity sub-asset classes weights are 60 percent and the sum of the fixed income sub-asset classes weights must equal 40 percent. The first optimization is run

[10] Credit, Government, and Inflation Linked Fixed Income.

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without additional constraints, in the second optimization a constraint of minimum one percent allocation in each sub-asset class is added and in the last optimization an additional constraint of maximum 20 percent allocation in one sub-asset class is added. The fifth and last portfolio is market capitalization weighted, where the sub-asset classes weights are weighted to sum to 60 percent and 40 percent, for public equity and fixed income, respectively.

Alternative risk measures

Standard deviation and the Sharpe ratio are both based on the assumption of normality of returns. However, there is a risk of deviations in the return data which can induce bigger risks (Bodie, 2018, p. 137). Therefore, further risk measures that account for non-normality will be deployed in this paper in order to study risk characteristics of the portfolio further; (1) skewness in order to measure the asymmetry of returns where a positive skew indicate domination of extreme positive values; (2) kurtosis which measures the deviation from normality, i.e. fatness of the tails of the distribution; (3) Sortino ratio which is an alternative performance measure to the Sharpe ratio. While the Sharpe ratio is the quotient of the risk premium and the standard deviation, the Sortino ratio uses the lower partial standard deviation (LPSD) of excess returns instead; (4) Value at Risk (VaR) which is a statistical risk measure commonly used amongst banks, treasury and portfolio managers. Using historical simulation computing monthly historical return data, the 95-percent VaR can be found in the 5^{th} percentile. Similarly to VaR the last alternative risk measure (5) Expected shortfall (ES) will be calculated. ES is the expected loss conditional on VaR (i.e. conditional on that we are in the tail of the distribution). This measure can be calculated as the average of the returns in the tail.

Portfolio evaluation

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After the construction of the optimal risky portfolios their performance will be evaluated through creating return series for each of the optimal portfolios, through weighting the returns of the proxies with the weights assigned in the portfolio optimization, on monthly basis. The returns of the optimal portfolios are benchmarked toward the similarly weighted return series of the identified market portfolio, through Jensen's alpha. Jensen's alpha provides a risk-adjusted performance measure of the optimal portfolios return relative the market portfolios return. This will indicate whether holding the optimal portfolio, as of 2020, would have over- or underperformed relative the market portfolio, accounting for market risk. The significance of the alphas is tested by a two-sided t-test at five percent significance level.

In order to derive whether there is any benefit to include private asset classes in a portfolio, the optimal portfolios will be benchmarked relative similarly constructed classical 60/40 stock-bond portfolio[11]. Outperformance is measured through three risk-adjusted return measures: Jensen's alpha, Sharpe ratio and Sortino ratio. Jensen's alpha shows the portfolios risk-adjusted performance based on systematic risk, as regressed on the market portfolio. The Sharpe ratio shows risk-adjusted return accounting for the portfolio risk (standard deviation) and the return on the risk-free rate (10-year US Treasury rates). Lastly, the Sortino ratio provides a risk-adjusted return estimate where normal distribution is not assumed, accounting only for negative returns.

Two factor analysis of variances (ANOVA) with replication are performed at five percent significance level, to see whether there is a significant difference between the Jensen's alpha, Sharpe ratio and Sortino ratio of the two samples (optimal risky portfolios and 60/40 stock-bond portfolios), five portfolios and two time periods (2014-2020 and 2003-2020).

[11] A common investing rule as well as portfolio benchmark has been the 60/40 balanced portfolio, consisting of 60 percent equity and 40 percent bonds. The idea of the strategy is to provide growth through the 60 percent equity allocation while dampening the volatility through the 40 percent bond allocation. Hence, the idea is stable growth and investing rule has been adopted by financial institutions such as pension funds and other investors for decades (Ambachtsheer, 1987; Bernsteinm, n.d.; Kaya, 2019; Kephart, 2021; McCormick, 2021; Toschi, 2021; Van Der Zwan, 2021). However, the investment rule has seen criticism in later years for being out-dated (Kaya, 2019; McCormick, 2021; Toschi, 2021). Allaria (2021) highlights the importance of the interest rate climate in regard to the strategy, as the low interest rates are likely to yield low future returns for the bonds.

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Robustness test

Various scholars have identified shortcomings with modern portfolio theory. The two major shortcomings regard the risk measurement and robustness of the model. Regarding risk measurement, Grootveld and Hallerbach (1999, p. 305) highlights that mean-variance optimization only result in an optimal portfolio given that returns are jointly elliptically distributed, since the variance does not treat positive and negative deviations from the mean differently. This implies that the returns must be symmetrically distributed. However, Mandelbrot (1963), Fama (1965) and Rachev et al. (2005) are some scholars who oppose to this assumption. In addition to asymmetry, extreme events are considered to have a higher probability of occurring relative than that predicted by the normal distribution (Rachev et al., 2005). Hence, other methods that account for asymmetry and extreme events, such as post-modern portfolio theory, which account for downside risk, have become popular. Regarding the robustness, Black and Litterman (1992), Jorion (1985) as well as Best and Grauer (1991) distinguish that small changes in expected returns are likely to yield large changes in optimal portfolio allocation and might result in a completely different optimal portfolio. Moreover, historical mean returns are only approximations for the expected returns when conducting mean-variance optimization. This will yield estimation errors, since historical means are not perfect approximations for the expected returns (Merton, 1980; Jorion, 1985; Michaud, 1989; Jagganathan & Ma, 2003). Therefore, some scholars have further modified the mean-variance optimization to account for robustness; Benati (2015) and Salah et al. (2016) use the sample median instead of the mean as robust statistics, as the median does not account for outliers (Delage and Ye 2010; Chen et al. 2011). Other approaches when minimizing portfolio risk has been to use value at risk (VaR) (see Fertis et al., 2012; Michaud, 2009; Poddig and Unger, 2012; Unger, 2015), conditional value-at-risk (see Huang et al. 2010), partitioned value-at-risk (see Goh et al. 2012), asymmetry-robust value-at-risk (see Natarajan et al. 2008), worst-case polyhedral value-at-risk (see Zymler et al. 2013), worst-case value-at-risk (see Huang et al. 2007) and worst-case

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quadratic value-at-risk (see Zymler et al. 2013). In regard to above mentioned reasons, it is of interest to study the robustness of the estimates.

To mitigate estimations errors and increase the validity of the results a robustness test is hence carried out, through bootstrapping the return data with resample. This implies creating new data sets with resample, based on the historical return data (Franke et al., 2004; Hull, 2018). This is in line with previous scholars (see Michaud, 2009; Poddig & Unger, 2012; Unger, 2015). In this paper, the return data for each portfolio and period will be resampled 100 times. This has a direct effect on the expected return and standard deviation of the portfolio. For each portfolio, the expected return, standard deviation, Sharpe ratio, LPSD and Sortino ratio is estimated. Doing this, best- and worst-case scenarios are demonstrated, along with the average and median for the 100 new samples. The range of these estimates, along with the average, indicates whether the original estimates are robust.

Moreover, using historical simulation a 95 percent confidence interval for the portfolios VaR is calculated based on the resampled data. This provides an estimate of VaR for any given month with 95 percent certainty.

3. Results

The market portfolio

The global market portfolio is showcased in Figure 1. The market portfolio demonstrates that most of the global market capitalization, almost 60 percent, is allocated in private real estate. The majority is, however, allocated within residential real estate (58.33 percent of the market portfolio, see Figure A1 in Appendix II). The subsequent largest asset classes are fixed income and public equity, with approximately 21 and 17 percent weight each, respectively. Figure A1 in Appendix II demonstrates the global market portfolio divided into further sub-asset classes and Figure A2 demonstrates the global market portfolio excluding private real estate. The total return

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of the 2020 market portfolio if holding the portfolio constant through 2014-2020 is about 60 percent in total, as shown in Figure 2. The sub-asset classes return data is showcased in Figure A1 in Appendix II. A full overview of the market portfolio and the market capitalization in USD can be seen in Table A2 in Appendix I.

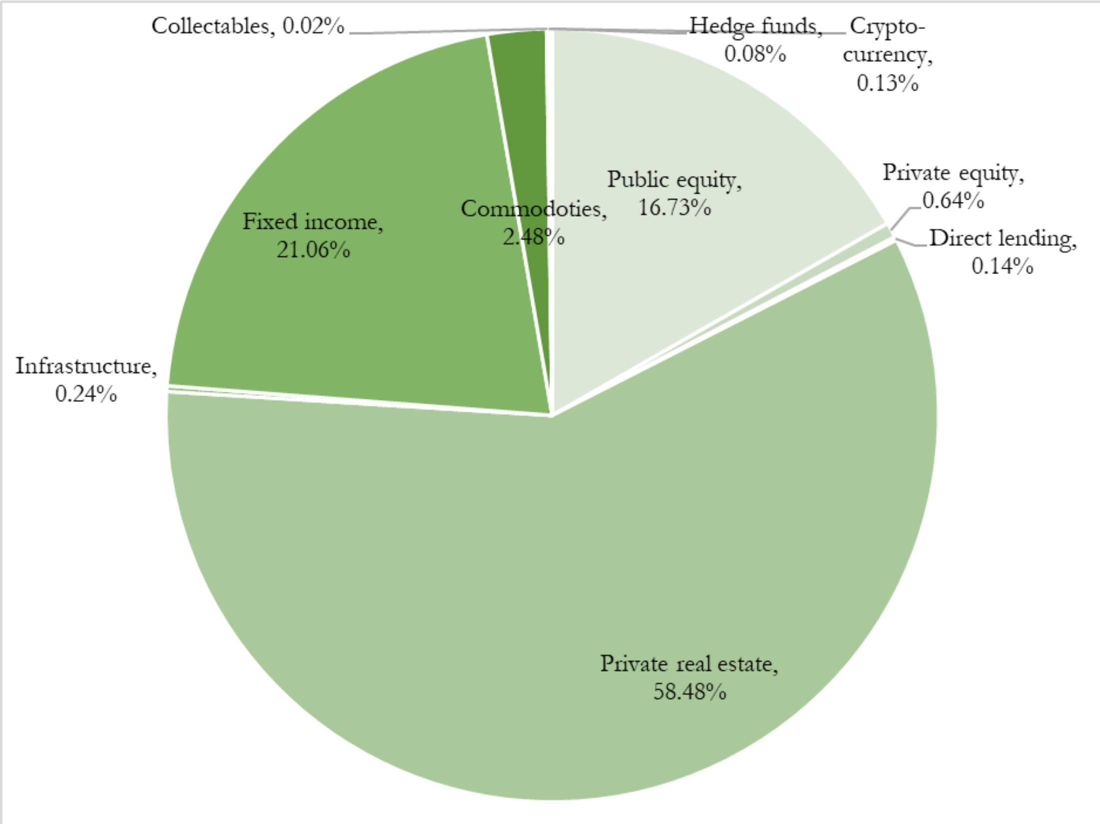

Figure 1. The market portfolio in percentage as of market capitalization in December 2020.

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Figure 2. Market portfolio index total return data between January 2014 and December 2020.

Markowitz portfolio optimization

Optimization 2014-2020: full data set

The Markowitz portfolio optimization is carried out in four steps to see the difference in optimal weights when adding constraints to the asset classes. Firstly, the expected return, standard deviation, Sharpe ratio, skewness and kurtosis is calculated for an equal weighted portfolio (see P1) consisting of all identified assets (see Table A1 in Appendix I). After conducting Markowitz optimization (see P2), the expected return of portfolio decreases by 0.77 percentage points while the risk decreases by 2.37 percentage points, and a Sharpe ratio of 24.5 is achieved. However, this optimization only suggests an investment in ten out of the 39 sub-asset classes with 76 percent invested in commercial real estate. The same Markowitz optimization is then conducted with a constraint of a minimum allocation of one percent in each sub-asset class (see P3). This portfolio yields higher expected return and risk while decreasing the Sharpe Ratio to 5.68. In P3 the optimal allocation in commercial real estate is almost 49 percent, which can still be considered very high from a diversification perspective. Hence, in P4 an additional constraint is added of a maximum 20 percent allocation in a single sub-asset class. This portfolio then yields a slightly higher expected

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return and risk than the previous, yet also a slightly lower Sharpe Ratio. However, the portfolio can be considered more diversified than previous portfolios. In the last portfolio, P5, minimum and maximum constraints are added to each asset class and sub-asset class, based on the asset and sub-asset classes weights in the global market portfolio (see Table A3 in Appendix I). This portfolio is the most diversified optimized portfolio, achieving a higher expected return while higher risk relative P2-P4. The Sharpe ratio has decreased to 4.75 in this portfolio, which is the lowest optimized Sharpe, however, it is still a high Sharpe ratio. The Sortino ratio moves similar as to the Sharpe, while ranging between 2.29 and 244.46. Moreover, considering VaR and ES, P2 demonstrates the best risk measures, which is likely due to the high allocations in sub-asset classes with low standard deviation. For P5 there is 95 percent certainty that the loss in any given month will not exceed 0.76 percent. Conditional on VaR, the expected loss (ES) in any given month is 1.61 percent with 95 percent certainty.

The optimizations yield very high allocations to especially commercial real estate but also real estate in general. A reasonable explanation is that all these sub-asset classes demonstrate a low standard deviation, especially commercial real estate, while still steady expected returns even if not as high as for other sub-asset classes. Important to keep in mind is that the real estate sub-asset classes commercial real estate, farmland and timberland, which get relatively high allocation, have data on quarterly level and have hence been exponentially smoothened. This could affect results depending on if volatility is captured in the estimates or not. Also art and private infrastructure equity have data on quarterly level, while not receiving as high weights which shows that it does not necessarily have an effect. However, private real estate is an illiquid asset class (Cheng et al., 2013; Chambers et al., 2018; BlackRock, 2020), why too high allocations are also not optimal. Ang (2014) show that the optimal weight of an asset class decreases the more illiquid the asset is. For instance, a weight of 59.3 percent in a perfectly liquid asset class will decrease to 37.3 percent if the asset class is traded on annual basis, and 25.1, 13.2 or 4.8 percent if the holding period is two, four or ten years, respectively (Ang, 2014). Therefore, due to the high allocation to commercial

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real estate, the same portfolio creation (equal weight and four-step optimization) is carried out when excluding this sub-asset class (38 indices remaining). This yields an aggregated optimal weight of 77.14 percent to the remaining private real estate sub-asset classes in P2xRCA, with farmland in the lead (see Table A6 in Appendix II). Also whisky, palladium, catastrophe bonds, private infrastructure equity were investments with relatively higher optimal weights occurring in more than two of the created portfolios. In P5xRCA the optimal weight of private real estate is the maximum relative its constraint, 30 percent. This is still a high allocation considering the illiquidity of the asset class. Hence, the portfolio creation is carried out excluding the whole private real estate asset class (35 indices remaining). In P2xRE to P4xRE the optimization yields higher weights to previously mentioned sub-asset classes, while in P5xRE more weight is allocated to public equity, fixed income, hedge funds and crypto currencies, relative previous optimizations (see Table A8 in Appendix II). In the optimization only excluding commercial real estate, similar Sharpe ratios are achieved for all portfolios except P2 and P2xRCA. P1xRCA-P5xRCA also yield slightly lower skew and higher kurtosis relative P1-P5. When excluding private real estate, the expected return in P1xRE-P5xRE increases as does the risk, which yields lower Sharpe ratios for all optimal portfolios (see Table A5 and Table A7 in Appendix II).

Furthermore, all portfolios have negative skewness except P2 and P2xRCA. The negative skew indicates that small gains can be expected frequently while large losses will occur more seldom. Hence, the portfolios are likely to yield stable profits, however, it is important to note the risk of large losses. The risk of large losses is captured by VaR and ES. Similarly, all portfolios except P2 and P2xRCA experience relatively high kurtosis in the first two sets, which indicates a risk for extreme outcomes, positive or negative, for the portfolios. While in the last set, excluding private real estate in full, the skew and kurtosis decrease. Figure 3 showcases the indexed total return of the portfolios P1-P5 if holding the optimal weights constant throughout the studied period, as for the identified market portfolio in Figure 2.

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Table 1. Expected return, standard deviation, Sharpe ratio, skewness, kurtosis and Sortino ratio for an equal weighted portfolio and four optimal portfolios, based on full data set of 39 sub-asset classes 2014-2020.

	(P1)	(P2)	(P3)	(P4)	(P5)
Expected return	5.36%	5.87%	6.65%	6.74%	7.14%
Standard deviation	2.37%	0.20%	0.95%	1.00%	1.29%
Sharpe ratio	1.88	24.26	6.04	5.80	4.83
Skewness	-1.22	0.45	-1.18	-1.18	-1.01
Kurtosis	5.88	0.41	5.44	5.42	4.69
LPSD	1.94%	0.02%	0.33%	0.36%	0.54%
Sortino ratio	2.29	244.46	17.38	16.02	11.53
95% VaR	-1.63%	0.31%	-0.32%	-0.40%	-0.76%
95% ES	-3.44%	0.19%	-1.09%	-1.18%	-1.61%

Notes: P1 = equal weighted portfolio, P2 = Markowitz optimal portfolio, P3 = Markowitz optimal portfolio with a constraint of minimum one percent allocation in each sub-asset class, P4 = Markowitz optimal portfolio with a constraint of minimum one percent and maximum 20 percent allocation in each sub-asset class, and P5 = Markowitz optimal portfolio with minimum and maximum constraint to every asset class and sub-asset class according to Table A3 in Appendix I.

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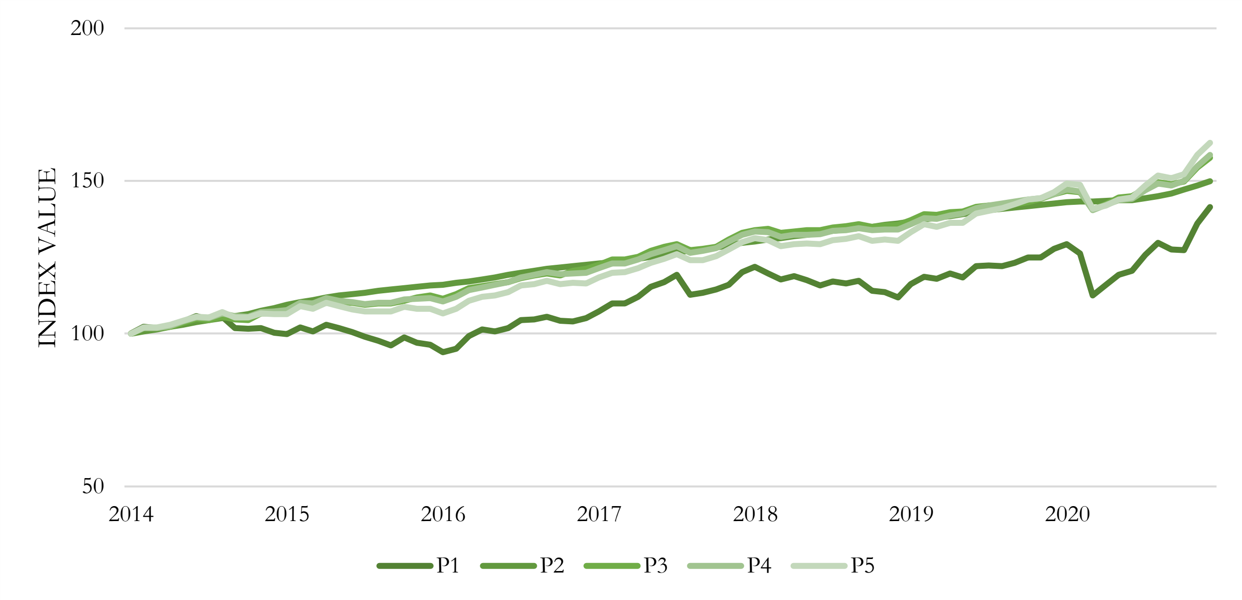

Figure 3. Index return series for portfolios P1-P5 (2014-2020), with base value 100 in January 2014.

Optimization 2003-2020: available data set

As the full data set could only be optimized for a seven-year period (2014-2020) the Markowitz portfolio optimization procedure is carried out again for a longer period (2003-2020). In these portfolios (P1*-P5*) a higher expected return, relative the previous portfolios (P1-P5), is achieved for all portfolios (see Table 2). Moreover, there is a higher risk in all portfolios but P5, measured by standard deviation. The Sharpe ratio is hence higher, for portfolios P1*, P3* and P4*, while lower for P2* and P5*, as the risk is much lower for P2 and a bit lower for P5 but similar expected return. The Sharpe ratios are still relatively high, spanning between 2.13 to 6.48. Similarly, the portfolios yield a lower and higher skewness and kurtosis, respectively. Studying the LPSD the risk of the portfolios has increased with the longer period studied, yielding Sortino ratios ranging between 2.10 to 17.16. Even if lower, these ratios can also be considered high. Also, the 95% VaR and ES have increased for all portfolios, indicating the risk of larger losses in any given month. The indexed total return series for the portfolios, P1*-P5*, for the period are shown in Figure 4.

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Table 2. Expected return, standard deviation, Sharpe ratio, skewness, kurtosis and Sortino ratio for an equal weighted portfolio and four optimal portfolios, based 30 sub-asset classes 2003-2020.

	(P1*)	(P2*)	(P3*)	(P4*)	(P5*)
Expected return	7.03%	7.45%	7.46%	6.92%	7.67%
Standard deviation	2.86%	1.01%	1.46%	1.74%	1.31%
Sharpe ratio	2.13	6.48	4.47	3.45	5.14
Skewness	-1.38	-1.09	-1.44	-1.57	-1.26
Kurtosis	6.45	6.40	7.13	7.69	5.52
LPSD	2.91%	0.38%	0.82%	1.20%	0.61%
Sortino ratio	2.10	17.16	7.94	5.00	11.12
95% VaR	-3.83%	-1.06%	-1.43%	-2.00%	-1.55%
95% ES	-7.41%	-1.93%	-3.25%	-4.17%	-2.89%

Notes: P1* = equal weighted portfolio, P2* = Markowitz optimal portfolio, P3* = Markowitz optimal portfolio with a constraint of minimum one percent allocation in each sub-asset class, P4* = Markowitz optimal portfolio with a constraint of minimum one percent and maximum 20 percent allocation in each sub-asset class, and P5* = Markowitz optimal portfolio with minimum and maximum constraint to every asset class and sub-asset class according to Table A3 in Appendix I.

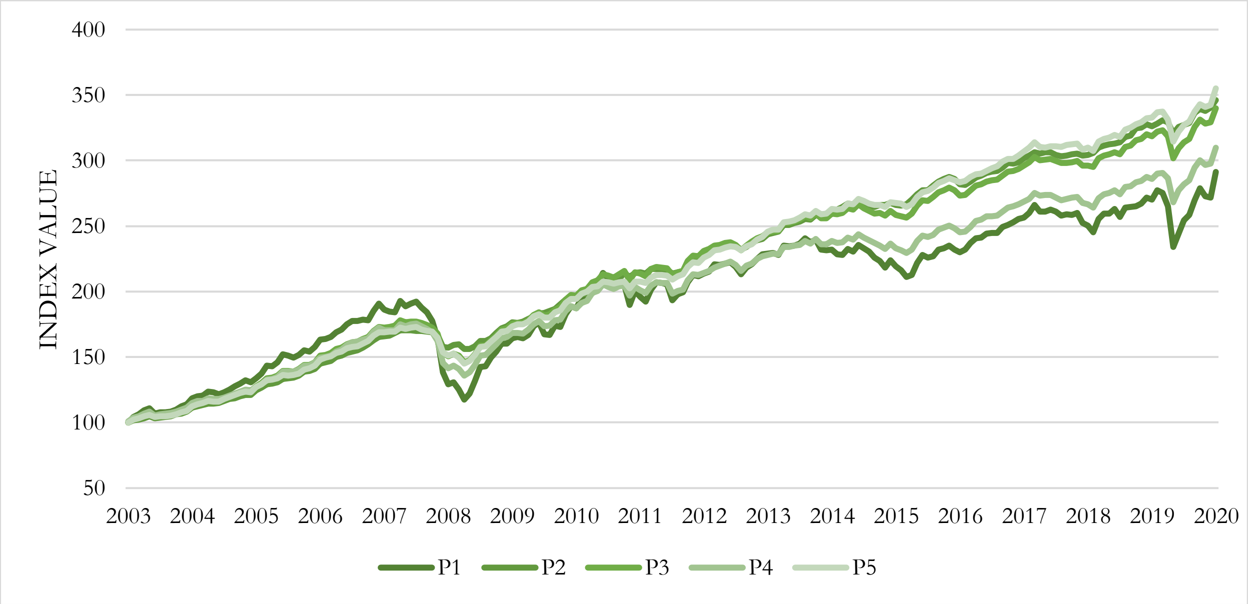

Figure 4. Index return series for portfolios P1*-P5* (2003-2020).

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Portfolio evaluation

In order to test whether private market investments benefit the optimal portfolios, five benchmark 60/40 stock-bond portfolios for the corresponding periods have been created. The same statistics as for the optimal risky portfolios are calculated for these portfolios, along with Jensen's alphas for all portfolios, and statistically compared to see whether there is a statistical difference.

Benchmark portfolios: 60/40 stock-bond portfolios

The first portfolio (M1) is, as previously explained, an equal weighted portfolio. In the first optimized 60/40 stock-bond portfolio (M2), 60 percent of the weight is given to North American public equity and 40 percent to government fixed income. Adding more constraints (M4) the allocation public equity from Asia Pacific and Frontier Markets are given more weight, along with credit (corporate and non-corporate) fixed income. The market capitalization weighted portfolio (M5) is the most diversified portfolio, with the third highest Sharpe ratio, having the same standard deviation as M2 that has the highest Sharpe ratio (see Table 3). See Table A9 in Appendix II for exact weight allocations.

Comparing the classical 60/40 stock-bond portfolios (see Table 3) with the optimal risky portfolios (see Table 1) for 2014-2020, lower risk-adjusted return, considering Sharpe and Sortino ratios, are found in all portfolios but M1. The range of the difference in Sharpe ratios is -0.23 to 21.23, with an average of 5.92. This implies that the Sharpe ratios for the optimal risky portfolios are on average 5.92 higher relative the 60/40 stock-bond portfolios. Important to note however is that the Sharpe of P2 substantially increases the average. While the expected return is higher for the 60/40 portfolios (on average by 1.61 percentage points), this is offset by higher risk as well (both standard deviation and LPSD by 1.50 and 1.03 percentage points on average, respectively). Similarly, the VaR and ES indicates a risk of larger losses in any given month with 95 percent certainty, in all portfolios. The skewness and kurtosis are higher and lower, respectively, for all 60/40 stock-bond portfolios, but M2.

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Table 3. Expected return, standard deviation, Sharpe ratio, skewness, kurtosis, LPSD, Sortino ratio, 95% VaR and 95% ES for five 60/40 stock-bond portfolios, 2014-2020.

	(M1)	(M2)	(M3)	(M4)	(M5)
Expected return	6.68%	9.04%	8.84%	7.21%	8.03%
Standard deviation	2.73%	2.68%	2.67%	2.57%	2.68%
Sharpe ratio	2.11	3.03	2.97	2.45	2.65
Skewness	-0.55	-0.20	-0.23	-0.66	-0.30
Kurtosis	3.12	2.12	2.22	4.06	2.33
LPSD	1.95%	1.46%	1.49%	1.86%	1.58%
Sortino ratio	2.96	5.55	5.32	3.39	4.50
95% VaR	-2.04%	-2.54%	-2.56%	-1.66%	-2.45%
95% ES	-4.03%	-4.02%	-4.01%	-3.73%	-4.14%

Notes: M1 = equal weighted portfolio, M2 = Markowitz optimal portfolio, M3 = Markowitz optimal portfolio with a constraint of minimum one percent allocation in each sub-asset class, M4 = Markowitz optimal portfolio with a constraint of minimum one percent and maximum 20 percent allocation in each sub-asset class, and M5 = Market capitalization weighted portfolio.

In the first and second optimization (M2* and M3*) of the 60/40 stock-bond portfolios in 2003-2020 similar weights as for 2014-2020 is retrieved. However, in optimization M4* further weight is given to public equity from Asia Pacific and emerging markets, along with inflation-linked fixed income, this is in addition to the weights of North American public equity and government fixed income. Important to note however is that frontier markets is not included in this period, as data is not available. See Table A10 in Appendix II for exact weight allocations.

Similar results as for 2014-2020 are obtained studying 2003-2020. The 60/40 stock-bond portfolios for 2003-2020 (see Table 4) all demonstrate lower Sharpe ratios by 2.05 on average, relative the optimal risky portfolios (see Table 2). The Sortino ratios are lower in all 60/40 stock-bond portfolios except M1*, by 5.90 on average. However, the expected return is on average 0.82 percentage points higher, and the standard deviation 1.49 percentage points higher. Also, in this

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period the 95% VaR and ES demonstrates that in any given month there is risk of a larger loss. The skewness and kurtosis are higher and lower, respectively, for all 60/40 stock-bond portfolios.

Table 4. Expected return, standard deviation, Sharpe ratio, skewness, kurtosis, LPSD, Sortino ratio, 95% VaR and 95% ES for five 60/40 stock-bond portfolios, 2003-2020.

	(M1*)	(M2*)	(M3*)	(M4*)	(M5*)
Expected return	8.01%	8.23%	8.22%	8.25%	7.92%
Standard deviation	3.44%	2.96%	2.98%	3.39%	3.07%
Sharpe ratio	2.06	2.47	2.45	2.16	2.28
Skewness	-0.84	-0.73	-0.75	-0.84	-0.82
Kurtosis	3.29	2.86	2.94	3.26	3.08
LPSD	3.21%	2.24%	2.29%	3.11%	2.50%
Sortino ratio	2.21	3.27	3.19	2.36	2.80
95% VaR	-5.64%	-4.91%	-4.87%	-5.27%	-4.89%
95% ES	-8.23%	-6.94%	-6.99%	-7.96%	-7.41%

Notes: M1* = equal weighted portfolio, M2* = Markowitz optimal portfolio, M3* = Markowitz optimal portfolio with a constraint of minimum one percent allocation in each sub-asset class, M4* = Markowitz optimal portfolio with a constraint of minimum one percent and maximum 20 percent allocation in each sub-asset class, and M5* = Market capitalization weighted portfolio.

Portfolio evaluation

The portfolio alphas, as showcased in Table 5, demonstrates portfolio outperformance relative the identified global market portfolio. 70 percent of the optimal risky portfolios have positive alphas, out of which 86 percent are significant at five percent significance level. Thus, the optimal risky portfolios have significantly outperformed the global market portfolio in 60 percent of the portfolios in both time periods, 2014-2020 and 2003-2020. The 60/40 portfolios have statistically negative alphas.

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Table 5. Alphas for each portfolio and sample, and its significance.

		1	2	3	4	5
Optimal	2014–2020	-0.47%***	0.46%***	0.24%***	0.21%**	0.11%
portfolios	2003–2020	-0.41%***	0.34%***	0.14%**	-0.02%	0.19%***
60/40-	2014–2020	-0.56%***	-0.32%	-0.34%**	-0.44%***	-0.44%***
portfolios	2003–2020	-0.52%***	-0.30%**	-0.31%**	-0.47%***	-0.39%***

Notes: Regression alphas (Jensen's alpha) for each sample (column 1), period (column 2) and portfolio (row 1), and its significance (two-sided t-test). Significance level: * = 10%, ** = 5% and *** = 1%.

Hereafter, this paper examines if there is a significant difference between the two samples (the optimal risky portfolios and the 60/40 stock-bond portfolios) alphas, Sharpe-, and Sortino ratios. The two factor ANOVA with replication shows that there is a statistically significant difference at five percent significance level between the alphas of the two samples, and between the different portfolios and the two time periods (see Table A11 in Appendix II). Similarly, the ANOVA show a statistical difference between the two samples Sharpe ratio. As shown in Tables 1 and 2 the Sharpe ratio of the optimal risky portfolios is higher than that off the 60/40 stock-bond portfolios in several portfolios (see Table A12 in Appendix II). Further, the ANOVA of the Sortino ratios does not show any significant difference between the samples, portfolios nor periods (see Table A13 in Appendix II). The results hence demonstrate that when the risk-adjusted return where market risk and portfolio risk are considered, Jensen's Alpha and Sharpe ratio, respectively, the performance of the optimal risky portfolios are significantly better relative the 60/40 stock-bond portfolios. Nevertheless, in the case where normal distribution is not considered and risk is measured as LPSD, as is the case with Sortino ratio, there is no significant difference between the two samples. Thus, in the case normal distribution is assumed the optimal risky portfolios outperforms 60/40 stock-bond portfolios, while if normal distribution is not assumed there is no overall significant outperformance of the optimal risky portfolios relative the 60/40 stock-bond

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portfolios. This highlights the outperformance of the classical 60/40 stock-bond portfolio if deploying the optimal risky portfolio including private and alternative asset classes. The results will be further discussed in section four,

Robustness test

The optimal portfolios (P1 to P5 and P1* to P5*) return data is resampled with replacement 100 times. This generates 100 different outcomes for each optimal portfolio (1 to 5) and period (2014-2020 and 2003-2020). The outcome shows that in a worst-case scenario the expected return could fall to 1.59 (5.21) percent for 2014-2020 (2003-2020). Similarly, in a best-case scenario the expected return could rise to 10.98 (10.09) percent for 2014-2020 (2003-2020), see Table 8 and Table 9. However, both the average and median expected return is similar to that of the original return data. The standard deviation and LPSD does not have a large range, however, the risk-adjusted return does. In no scenario or period is the Sharpe nor Sortino ratio negative, but in the best-case scenario the ratios reached a maximum of 10.09 (9.41) and 80.22 (51.89), respectively, for 2014-2020 (2003-2020). Notably, the average Sortino ratio for P5 (2014-2020) is more than double that of the original estimate.

The resampled data hence indicates the risk of different outcomes to that expected in the original estimates. However, the averages and medians are mainly centred around these original estimates. For 2014-2020 the resample indicates slightly better estimates, while for 2003-2020 the resample indicate slightly worse estimates. Hence, the estimates can be assumed robust on average.

Moreover, the 95 percent confidence interval for VaR is -0.76 to -5.37 (-1.08 to -5.05) percent for any given month for P5 (P5*) in 2014-2020 (2003-2020). Table 6 demonstrate the values for P5, Table 7 for P5*, then Table A14 and A15 in Appendix II demonstrates the values for P1-P4 and P1*-P4*, respectively.

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Table 6. Descriptive statistics based on robustness test (resample with replacement) for optimal portfolio P5 (covering the period 2014-2020).

	Min	Max	Average	Median
Expected return	1.59%	10.98%	7.22%	7.23%
Standard deviation	0.94%	1.94%	1.25%	1.21%
Sharpe ratio	0.38	10.09	5.28	5.36
LPSD	0.12%	1.22%	0.45%	0.53%
Sortino ratio	0.60	80.22	24.65	14.45

Notes: Expected return, standard deviation, Sharpe ratio, LPSD and Sortino ratio is calculated for each of the 100 portfolios. Table 8 display the minimum, maximum, average and median of these statistics. See Table A10 in Appendix II for P1 to P4.

Table 7. Descriptive statistics based on robustness test (resample with replacement) for optimal portfolio P5* (covering the period 2003-2020).

	Min	Max	Average	Median
Expected return	5.21%	10.09%	7.52%	7.59%
Standard deviation	0.97%	1.64%	1.32%	1.33%
Sharpe ratio	2.90	9.41	5.09	4.92
LPSD	0.18%	1.03%	0.59%	0.61%
Sortino ratio	4.82	51.89	13.23	10.45

Notes: Expected return, standard deviation, Sharpe ratio, LPSD and Sortino ratio is calculated for each of the 100 portfolios. Table 8 display the minimum, maximum, average and median of these statistics. See Table A11 in Appendix II for P1* to P4*.

4. Discussion

Global market portfolio

The weights of the global market portfolio (as of December 2020) are to some parts roughly in line with previous findings. However, comparisons to previous studies are difficult, as

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different studies have included different asset classes. For example, this study includes private real estate (such as residential, commercial and agricultural), which is the largest asset class considering market capitalization, decreasing the weight of all other asset classes both in absolute terms in this study but also relative other studies as the majority of other studies exclude these sub-asset classes. However, Savills (2017) also include private real estate (residential, commercial, and agricultural etc.) and similar estimates as to equities (private and public), and fixed income (see Table 8). This explains why the other studies demonstrate higher weights to e.g. public equity and fixed income. If excluding (private) real estate (public real estate equity is still included in public equity) from this study (see column This study x RE in Table 8), the weights of public equity and fixed income are similar to the estimates of Robeco (2020). However, the estimates of cryptocurrencies are lower, even if the estimate of Robeco (2020) only include Bitcoin. This is likely since cryptocurrencies, especially Bitcoin, have had a large increase during 2020. However, relative to other studies this study's estimate of private equity seems relatively low, even when (private) real estate is excluded.

Another interesting note is that real estate is a highly leveraged asset class, which increases the market capitalization. A higher leverage ratio further increases the financial risk of the asset class, which is not included in the normal risk statistics, such as standard deviation.

For further information about the identified market capitalizations, the proxies used and comparisons to other estimates and reports see Appendix IIII.

Table 8. The market weights of broad asset classes, as identified in this study, by Robeco (2020), Doeswijk et al. (2020), Savills (2017), Gadzinski et al. (2016), Doeswijk et al. (2014) and Ibbotson & Sigel (1983).

	This study	**This study x RE**	**Robeco (2020)**	**Doeswijk et al. (2020)**	**Savills (2017)**	**Gadzinski et al. (2016)**	**Doeswijk et al. (2014)**	**Ibbotson & Sigel (1983)**
Public equity	16.73%	40.29%	40.80%	53.40%	17.00%	13.00%	36.30%	11.30%
Private equity	0.64%	1.54%	4.70%	-		20.00%	3.60%	

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Real estate	58.48%	-	4.50%	3.90%	56.00%	22.00%	5.10%	52.50%
Infrastructure	0.24%	0.58%	-	-	-	-	-	-
Fixed income	21.06%	50.72%	49.40%	39.40%	25.00%	37.00%	55.00%	14.10%
Cryptocurrencies	0.13%	0.31%	0.60%	-	-	-	-	-
Commodities	2.48%	5.97%	-	3.30%	2.00%	-	-	7.80%
Cash	-	-	-	-	-	8.00%	-	2.10%
Other	0.24%	0.58%	-	-	-	-	-	12.00%

Notes: Asset class specifications: This study: Real estate = only private real estate, Other = Hedge funds + Collectables + Direct lending. Robeco: Cryptocurrencies = Gold. Savills: Commodities = Gold. Gadzinski et al.: Cash = Cash & cash equivalent, Real estate = Real estate + Land. Ibbotson & Sigel: Commodities = Metals, Other = Durables.

This study = Identified market weights in this study. This study x RE = The market weights of each asset class if "Real estate" (see Private Real Estate in Table 1 in Appendix I) is excluded.

Optimal allocations

Optimal risky portfolio performance

The results from the analysis demonstrates enhanced portfolio performance in regard to Jensen's alpha and Sharpe ratio of the constructed optimal risky portfolios relative the 60/40 stock-bond portfolios constructed, significant at five percent significance level. The Sharpe ratios for the optimal risky portfolios 2014-2020 (2003-2020) were relatively high, ranging between 1.88 and 24.26 (2.13 and 6.48) with an average of 8.56 (4.33). While still high for the 60/40 stock-bond portfolios 2014-2020 (2003-2020), ranging between 2.11 and 3.03 (2.06 and 2.47) with an average of 2.64 (2.29), the ratios are significantly lower. Similarly, the risk (standard deviation, LPSD, VaR and ES) are higher for all 60/40 stock-bond portfolios both in 2014-2020 and 2003-2020. This demonstrates the potential benefits of including alternative and private asset classes into portfolio construction, increasing the risk-adjusted return. This is of interest, as the 60/40 stock-bond investing strategy have been commonly employed by financial institutions (see 2.5. 60/40 stock-

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bond portfolios). The strategy has, however, faced criticism in later years and been called for an update, which can be considered in line with the results of this paper regarding the outperformance of the constructed optimal portfolios. The analysis also demonstrates very high Sortino ratios for the optimal risky portfolios in 2014-2020 and 2003-2020, while especially for 2014-2020. Nevertheless, there is no significant difference of the Sortino ratios relative the 60/40 stock-bond portfolios.

Another interesting finding is the lower Sharpe ratios and Sortino ratios when studying 2003-2020 relative to 2014-2020. For the optimal risky portfolios all Sortino ratios and three of five Sharpe ratios were lower and in 2003-2020 relative 2014-2020. Similarly, for the 60/40 stock-bond portfolios all Sortino and Sharpe ratios were lower in 2003-2020. There are various possible explanations for this. One explanation is that the risk (expected return) was higher (lower) in this period, which is reasonable considering that for example the global financial crisis between 2007-2008 is included in the data for 2003-2020. Another explanation is the exclusion of sub-asset classes in the longer period, due to lack of data availability. Sub-asset classes such as whisky and cryptocurrencies are assets not included in this period, however, they have demonstrated high returns and little correlation with other asset classes, hence given a relatively larger weight than other assets in the optimization.

Moreover, according to EMH the market is efficient and that is why the global market portfolio should showcase the efficient allocations of assets. However, Jensen's alpha show that the optimal risky portfolios also outperform the market portfolio, while the 60/40 stock-bond portfolios perform worse, when systematic risk is accounted for. This implies that the optimal risky portfolios have a more efficient allocation on a risk-adjusted basis, relative to the actual funds allocated which are considered efficient according to EMH. These findings further indicate the potential benefit of private market asset classes inclusion, both in portfolio optimization but also in market portfolio composition. Firstly, the optimal risky portfolios perform better in terms of alpha relative both the global market portfolio but also the 60/40 stock-bond portfolios. Secondly,

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the identified global market portfolio can be considered a better benchmark when measuring portfolio performance (as of the 60/40 stock-bond portfolios), as simply benchmarking these portfolios to a broad public equity index does not demonstrate performance relative the market, but simply relative public equity.

Optimal asset classes

In regard to optimal asset classes, private real estate has shown to be receive high optimal weight allocations in every optimization and period), along with fixed income, commodities and collectables.

Studying the optimal weights of the optimal risky portfolios 2014-2020 commercial real estate, residential real estate, timberland real estate, farmland, private infrastructure equity, securitized and CAT bonds, palladium, whisky and North American public equity are sub-asset with relatively higher weights. Similarly, in the 2003-2020 optimal risky portfolios the same sub-asset classes are allocated high weights along with Asia Pacific public equity, all fixed income sub-asset classes, gold, art, wine and hedge funds. Important to note is that sub-asset classes as whisky and palladium which are allocated high weights in 2014-2020 are not included due to data availability. Lastly, in the optimal risky portfolios 2014-2020 excluding private real estate as an asset class the following sub-asset classes are considered optimal to allocate higher weights to: North American, Asian Pacific and European public equity, indirect private equity, private infrastructure equity, credit and government bonds excluding emerging markets, global securitized bonds, CAT bonds, gold, palladium, collectable whisky and cars, cryptocurrencies, and hedge funds.

If counting the frequency of when the sub-asset classes receive a weight of more than 0.5 percent relative the minimum constraint (zero for un-constrained) in the optimal risky portfolios for 2014-2020, 2014-2020 excluding private real estate and 2003-2020, some sub-asset classes occur more frequently (see Figure 5). The sub-asset classes having a higher frequency than the average frequency (rounded 4.5) are: North American public equity, private commercial,

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residential, farmland and timberland real estate, private infrastructure equity, government bonds, securitized bonds, CAT bonds, palladium, whisky and hedge funds.

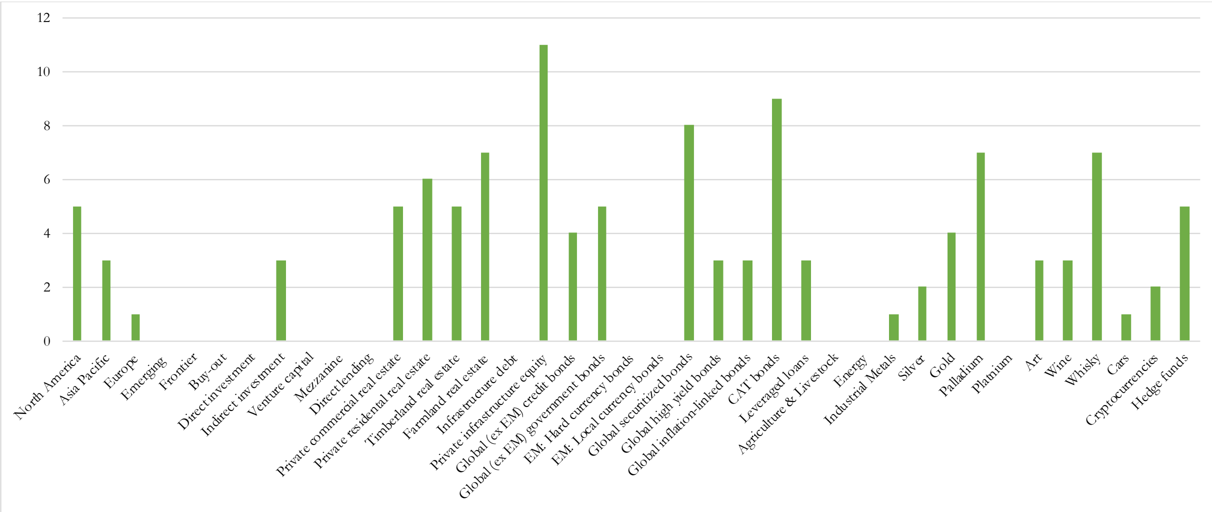

Figure 5. Frequency of times the sub-asset classes are allocated a weight of more than 0.5 percent of the minimum constraint (zero if no constraint).

The results hence show that collectables (especially whisky) are a good sub-asset class to invest in (holding these portfolios), which is not as common to invest in as for example public or private equity, fixed income or private real estate. In terms of commodities, palladium receives highest allocation, higher than gold, which is a more common asset to invest in, especially as inflation hedge. Relatively, cryptocurrencies did not yield high weights nor frequency. Even while yielding high expected return, cryptocurrency demonstrate high volatility. This is further of interest as cryptocurrencies is currently a highly debated investment. The optimal weight ranges between 0.05 to 2.24 percent, and according to Robeco (2020) research shows that an optimal weight of bitcoin (note: not cryptocurrencies) in a portfolio is 1 percent.

Nevertheless, important to note is that private markets are relatively illiquid (Cheng et al., 2013; Chambers et al., 2018; BlackRock, 2020). Cheng et al. (2013) quantifies the liquidity risk from investing in private markets, through focusing on commercial real estate. The scholars find that the liquidity risk is important to account for, especially in the case of down markets, a case when

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liquidity risk becomes a greater concern. In line with this, Ang (2014) demonstrates that the optimal weights of an asset class decreased much with an increase of the holding period (illiquidity) of the asset. As the optimization procedure does not account for illiquidity but only expected return, standard deviation and covariances, this causes implications. Further, real estate is a highly leveraged asset class relative other asset classes, as previously mentioned. This adds financial risk to the asset, which the risk statistics of this paper do not cover. Therefore, the asset class is likely to retrieve higher weights than the optimal, if this risk would be accounted for. The results yield high weights in private market asset classes, as seen in the first optimization without any constraints, as in subsequent ones. Therefore, the final optimizations (portfolios four and five) can be argued to be more reasonable, as they provide further diversification (as Markowitz (1952) also highlight the importance of) and decrease the maximum weights in one asset class, which is shown especially important for illiquid asset classes, such as private real estate. For future studies it would be interesting, and relevant, to correct for illiquidity premiums (scholars have studied and stated the existence of these premiums in these illiquid asset classes, see Leitner et al., 2007; Cheng et al., 2013; Markwat et al., 2015; Netspar, 2019) and see if the optimal weights of the asset classes and sub-asset classes change, yielding different results.

Optimal risky portfolio

The creation of the optimal risky portfolio of the identified asset classes differ depending on the time period studied as well as which asset classes are considered in the analysis, as the weights are assigned considering expected return, standard deviation and the correlation between the assets. The results demonstrate that the Sharpe and Sortino ratio decrease as constraints and diversification increase, nevertheless, the ratios are still considerably high indicating good portfolio performance. Thus, portfolio four and five of every optimization can be argued as more realistic portfolio strategies as more diversified, minimizing the liquidity risk as high weights to private assets decrease. Between these two portfolios, portfolio five is further diversified, as consisting of

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more narrow constraints accounting for the assets actual weight in the market. This paper hence suggests portfolio five as the most optimal, as it still showcase high risk adjusted return while being further diversified than the others.

If considering all identified asset classes (see Table A1 in Appendix I) the optimal portfolio should be constructed as P5 (see left pie chart in Figure 6 or Table A4 in Appendix II). The largest constituent is private real estate (30 percent) which also is the largest part of the identified market portfolio, even if given a less weight in this portfolio. Similarly, Bekkers et al. (2009) find that the optimal weight to real estate is 25.7 percent. The optimization by Bekkers et al. also yields a weight of 28.6 percent to bonds, 26.4 percent to stocks, 12.7 percent to commodities and 6.6 percent to high yield (while zero percent to private equity, hedge funds, credits, and inflation-linked bonds). Nevertheless, previous optimized portfolios in this paper (see P2 to P4) suggest an optimal weight of private real estate ranging between 91.52 and 49.49 percent. In its Survey of Consumer Finances (SCF) the Federal Reserve shows that the percentage holding in primary residence of Americans is 64.9 percent in 2019. This is a 1.2 percentage point increase relative 2016 (Federal Reserve Bulletin, 2020). Causa et al. (2019) studies OECD countries and shows that housing is the largest constituent of households' portfolios. Residential real estate represents approximately 50 percent of the assets on average across the countries, ranging between 25 to 70 percent. Benjamin et al. (2004) studies the paradox of the high household weights in housing while relatively small weights in financial assets, in America, and finds that household tend to concentrate their holdings in real estate given a higher marginal propensity to consume in housing relative financial assets. The research therefore demonstrates existing high weights to private real estate for an average household, due to owning residential. Further, while all other assets receive a much higher weight than its market weight (relatively not in absolute terms), fixed income is only given a slightly higher allocation relative its market weight and public equity receives a lower weight.

However, as previously been discussed in this paper, it is also interesting to exclude private real estate from the optimal portfolio. This can especially be argued for as household weight to

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real estate can be considered fixed due to residential reasons, e.g. if not possible to increase or decrease the holdings as it could affect ones residential solutions. Also, Causa et al. (2019) and the Federal Reserve Bulletin (2020) have shown already high allocations in residential for households. The P5xRE optimal portfolio yields much higher optimal weights to fixed income, public equity, commodities, cryptocurrencies and hedge funds (see right pie chart in Figure 6 or Table A8 in Appendix II).

Both portfolios differ greatly to the classical 60/40 stock-bond portfolio, even though the weight to fixed income increases to 30 percent if excluding private real estate. The optimal weight of equities is much lower than the 60 percent allocation since risk-adjusted return is enhanced as more low-correlation assets are included that still demonstrate steady expected returns and lower risk. The results of the optimal portfolio also demonstrate the benefit of investing in not so common asset classes (e.g. collectables) or increasing the weight in these asset classes. Important to note is that the these are the optimal allocations found in a well-diversified portfolio, where all asset classes also consider of various sub-asset classes, and where illiquidity has not been taken into account more than maximum constraints of asset classes and sub-asset classes.

Conclusively, P5 (including real estate) can be seen as an optimal portfolio from a more institutional investment point of view, where private real estate is a part of a larger portfolio. However, P5xRE can be seen as an optimal portfolio if real estate investments are already fixed, excluding this asset from the portfolio. This can further be argued for keeping in mind that the statistical risk estimates in this paper likely underestimate the true risk of this asset class, as the return was provided at quarterly level, and as the asset tend to be highly illiquid and leveraged increasing the financial risk.

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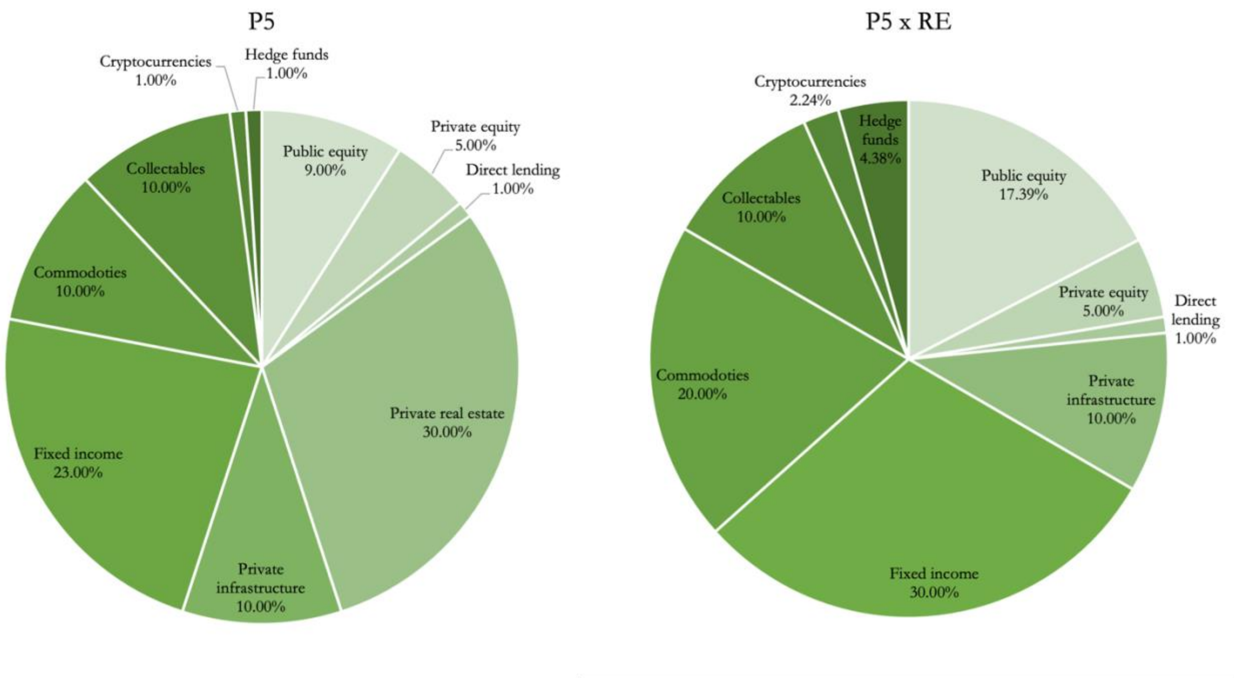

Figure 6. Optimal risky portfolios including (left pie chart) and excluding (right pie chart) private real estate, 2014-2020.

5. Conclusion

The aim of this study is to further update the global market portfolio through adding asset classes and studying the asset classes in further detail in recent time, along with studying the optimal allocations of the identified asset and sub-asset classes relative the identified global market portfolio and 60/40 stock-bond portfolios, a common investment strategy deployed by financial institutions. This paper finds that the market weights of the identified market portfolio highly depend on which asset classes and sub-asset classes are included; while the largest allocation is to private real estate assets, many studies have excluded these, as well as other constituents. The results from the analyses demonstrates that the optimal risky portfolios outperform both the identified global market portfolio and the 60/40 stock-bond portfolios, at five percent significance level. This demonstrates the benefit of including private asset classes in portfolio composition. Nevertheless, private asset classes such as private real estate and certain collectables yield very high weights in certain optimizations carried out, which is not feasible considering the illiquidity of the

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assets, as well as the importance of diversification. Adding minimum and maximum constraints in the optimization either in absolute terms or in regard to the assets market weight can therefore be a good option in to create a well-diversified portfolio, not consisting of only a few illiquid asset classes with high weights. However, certain investments recurringly yield high weights which can be interesting to study further in case of portfolio construction where these have not yet been included. Important to keep in mind is the risk of estimation errors when using proxies, historical data and private markets data. For future papers it would be of interest to study the optimal weights of identified asset classes and sub-asset classes along with the benefit of these while accounting for illiquidity and potentially financial risk (especially in regard to real estate). The findings of this paper add to the existing literature and brings informational value in terms of a more detailed and updated proxy for the market portfolio, and the potential benefit of including alternative asset classes in portfolio construction.

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Appendix I. Data sources and market capitalization

Table A1. Identified asset classes and sub-asset classes, proxies used for each sub-asset class, its total return data level, the first date of index data.

Asset class	Sub-asset class	Index name	Data level	First date	Source
Public equity	North America	MSCI North America IMI	Monthly	1994-05-31	Bloomberg
	Asia Pacific	MSCI AC Asia Pacific IMI	Monthly	1995-01-31	Bloomberg
	Europe	MSCI Europe IMI	Monthly	1994-05-31	Bloomberg
	Emerging markets	MSCI Emerging Markets IMI	Monthly	1995-01-31	Bloomberg
	Frontier markets	MSCI Frontier Markets IMI	Monthly	2010-12-31*	Bloomberg
Private equity	Buy-out	LPX Buy-Out Index	Monthly	1999-01-29	Direct
	Direct investment	LPX Direct Index	Monthly	1998-12-31	Direct
	Indirect investment	LPX Indirect Index	Monthly	1999-12-31	Direct
	Venture capital	LPX Venture Capital Index	Monthly	1999-01-29	Direct
	Mezzanine	LPX Mezzanine Index	Monthly	2003-11-28	Direct
Direct lending	Direct lending	DLX Direct Lending Index	Monthly	2009-01-30*	Direct
Private Real Estate	Private commercial real estate	RCA CPPI Index	Quarterly	2000-12-29	Bloomberg
	Residential real estate	RPPI (OECD)	Monthly	2000-01-01	OECD[12]
	Timberland	NCREIF Timberland Index	Quarterly	1987-03-31	Direct
	Farmland	NCREIF Farmland Index	Quarterly	1991-04-30	Direct
	Infrastructure debt	Dow Jones Brookfield Global Infrastructure Broad Market Corporate Bond	Monthly	2011-01-31*	SP Global[13]

[12] https://stats.oecd.org/
[13] https://www.spglobal.com/spdji/en/indices/fixed-income/dow-jones-brookfield-global-infrastructure-broad-market-corporate-bond-index/#overview

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Private infrastructure	Infrastructure equity	Infra300 Equity Index	Quarterly	2000-06-30	EDHEC[14]
Fixed income	Global Non-corporate + Corporate x EM (inc EM)	Bloomberg Barclay Global Aggregate Credit ex Emerging Markets (Bloomberg Barclay Global Aggregate Credit)	Monthly (Monthly)	2014-01-31** (2000-09-29)	Bloomberg (Bloomberg)
	Global Government x EM (inc EM)	Bloomberg Barclay Global Aggregate Government ex Emerging Markets (Bloomberg Barclay Global Multiverse Government)	Monthly (Monthly)	2013-03-29** (2001-01-21)	Bloomberg (Bloomberg)
	EM: Hard Currency	Bloomberg Barclays Emerging Markets Hard Currency Aggregate	Monthly	2001-08-31*	Bloomberg
	EM: Local Currency	Bloomberg Barclays Emerging Markets Local Currency Government	Monthly	2008-06-30*	Bloomberg
	Global: Securitized	Bloomberg Barclays Global Aggregate Securitized Total Return Index	Monthly	2000-09-29	Bloomberg
	Global High Yield	Bloomberg Barclay Global High Yield Total Return Index	Monthly	1990-01-31	Bloomberg
	Global (ex EM) Inflation-linked	Bloomberg Barclays Global Inflation-Linked	Monthly	1997-10-31	Bloomberg
	Catastrophe Bond	Swiss Re Cat Bond Index	Monthly	2002-01-31	Eikon
	Leveraged Loans	Credit Suisse Leveraged Loan	Monthly	1991-12-31	Bloomberg
Commodities	Agriculture & Livestock	S&P GSCI Agriculture & Livestock	Monthly	1970-02-27	Bloomberg
	Energy	S&P GSCI Energy	Monthly	1983-01-31	Bloomberg
	Industrial Metals	S&P GSCI Industrial Metals	Monthly	1977-01-31	Bloomberg
	Precious Metals	S&P GSCI Silver	Monthly	1973-01-31	Bloomberg
		S&P GSCI Gold	Monthly	1978-01-31	Bloomberg
		S&P GSCI Palladium	Monthly	2008-09-30*	Bloomberg
		S&P GSCI Platinum	Monthly	1985-11-29	Bloomberg
Collectables	Art	ArtPrice Global Index	Quarterly	1998-01-01	ArtPrice[15]

[14] https://indices.edhecinfra.com/
[15] https://www.artprice.com/artmarketinsight/art-market-barometer

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	Wine	Liv-ex Fine Wine 100	Monthly	2001-07-31	Bloomberg
	Whisky	World Whisky Index	Monthly	2013-01-31*	Direct
	Cars	Hagerty Blue Chip Index	Monthly	2007-01-31*	Bloomberg
Crypto currency	General	Bloomberg Galaxy Crypto Index Bitcoin closing price	Monthly (Monthly)	2017-08-31* (2013-10-01)	Bloomberg (CoinDesk[16])
Hedge funds	General	Eurekahedge Hedge Fund Index	Monthly	1999-12-31	Bloomberg

Notes: EM = Emerging markets, Ex = Excluding. All data on quarterly level have been exponentially smoothed to monthly data level. * = Not included in 2003-2020 analysis, ** = Index in brackets used for 2003-2020 analysis, Direct = email contact directly with company provider. Observe that source only applies for return data and not market capitalization. For further information regarding proxies see Appendix III.

Table A2. Market capitalization and weights for each asset class and sub-asset class in billion USD.

Asset class	**Sub-asset class**	**Market cap**	**Market cap**	**Weight**	**Weight**
Public equity	North America	63 597.29	101 874.20	10.44%	16.73%
	Asia Pacific	23 132.00		3.80%	
	Europe	14 830.32		2.43%	
	Emerging	152.17		0.02%	
	Frontier	162.43		0.03%	
Private equity	Buy-out	1 740.45	3 870.72	0.29%	0.64%
	Direct investment	1 758.44		0.29%	
	Indirect investment	1.68		0.00%	

[16] https://www.coindesk.com/price/bitcoin

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	Venture capital	162.54		0.03%	
	Mezzanine	207.61		0.03%	
Direct lending	Direct lending	869.08	869.08	0.14%	0.14%
Private real estate	Private commercial real estate	1 182.29	356 203.11	0.19%	58.48%
	Residential real estate	354 966.39		58.28%	
	Timberland	35.24		0.01%	
	Farmland	19.20		0.00%	
Private infrastructure	Infrastructure debt	1 194.41	1 440.23	0.20%	0.24%
	Infrastructure equity	245.82		0.04%	
Fixed income	Global (ex EM) Non-corporate + Corporate	25 191.50	128 300.00	4.14%	21.06%
	Global (ex EM) Government	60 256.03		9.89%	
	EM: Hard Currency	4 875.81		0.80%	
	EM: Local Currency	8 079.51		1.33%	
	Global: Securitized	15 222.55		2.50%	
	Global High Yield	5 296.78		0.87%	
	Global (ex EM) Inflation-linked	5 627.83		0.92%	
	CAT Bond	1 300.10		0.21%	
	Leveraged Loans	2 449.88		0.40%	
Commodities	Agriculture & Livestock	324.46	15 116.39	0.05%	2.48%
	Energy	2 049.93		0.34%	
	Industrial Metals	514.00		0.08%	

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	Silver	1 409.00		0.23%	
	Gold	10 801.00		1.77%	
	Palladium	10.00		0.00%	
	Platinum	8.00		0.00%	
Collectables	Art	64.12	123.94	0.01%	0.02%
	Wine	0.14		0.00%	
	Whisky	59.00		0.01%	
	Cars	0.68		0.00%	
Cryptocurrencies	General	766.00	766.00	0.13%	0.13%
Hedge funds	General	507.61	507.61	0.08%	0.08%

Notes: For further information about the construction and sources of market capitalization, see Appendix IIII.

Table A3. Portfolio optimization weight caps.

Market cap (Sub-asset class)	**Min**	**Max**	**Market cap (Asset class)**	**Min**	**Max**
0-5%	1%	10%	0-5%	1%	10%
6-10%	2.5%	15%	6-10%	5%	20%
+11%	5%	20%	+11	5%	30%

Notes: EM = Emerging markets, Ex = Excluding. All data on quarterly level have been exponentially smoothed to monthly data level. For further information about the construction and sources of market cap, see Appendix IIII.

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Appendix II. Results from analysis

Table A4. Optimal weights for each asset class and sub-asset class from the portfolio optimization, 2014-2020.

Asset class	**Sub-Category**	**(1)**		**(2)**		**(3)**		**(4)**		**(5)**	
Public equity	North America	12.82%	2.56%	0.00%	0.00%	5.00%	1.00%	5.00%	1.00%	9.00%	5.00%
	Asia Pacific		2.56%		0.00%		1.00%		1.00%		1.00%
	Europe		2.56%		0.00%		1.00%		1.00%		1.00%
	Emerging		2.56%		0.00%		1.00%		1.00%		1.00%
	Frontier		2.56%		0.00%		1.00%		1.00%		1.00%
Private equity	Buy-out	12.82%	2.56%	0.00%	0.00%	5.00%	1.00%	5.00%	1.00%	5.00%	1.00%
	Direct investment		2.56%		0.00%		1.00%		1.00%		1.00%
	Indirect investment		2.56%		0.00%		1.00%		1.00%		1.00%
	Venture capital		2.56%		0.00%		1.00%		1.00%		1.00%
	Mezzanine		2.56%		0.00%		1.00%		1.00%		1.00%
Direct lending	Direct lending	2.56%	2.56%	0.00%	0.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Private real estate	Private commercial real estate	10.26%	2.56%	91.52%	76.04%	58.12%	1.00%	49.49%	15.55%	30.00%	1.00%
	Residential real estate		2.56%		2.56%		7.36%		12.94%		18.00%
	Timberland		2.56%		5.46%		1.00%		1.00%		1.00%
	Farmland		2.56%		7.46%		48.76%		20.00%		10.00%
	Infrastructure debt	5.13%	2.56%	0.00%	0.00%	2.40%	1.00%	9.03%	1.00%	10.00%	1.00%

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Private infrastructure	Infrastructure equity		2.56%		0.00%		1.40%		8.03%		9.00%
Fixed income	Global (ex EM) Non-corporate + Corporate	23.08%	2.56%	7.02%	0.00%	9.00%	1.00%	9.00%	1.00%	23.00%	1.00%
	Global (ex EM) Government		2.56%		0.00%		1.00%		1.00%		2.50%
	EM: Hard Currency		2.56%		0.00%		1.00%		1.00%		1.00%
	EM: Local Currency		2.56%		0.00%		1.00%		1.00%		1.00%
	Global: Securitized		2.56%		3.77%		1.00%		1.00%		4.50%
	Global High Yield		2.56%		0.00%		1.00%		1.00%		1.00%
	Global (ex EM) Inflation-linked		2.56%		0.00%		1.00%		1.00%		1.00%
	CAT Bond		2.56%		3.25%		1.00%		1.00%		10.00%
	Leveraged Loans		2.56%		0.00%		1.00%		1.00%		1.00%
Commodities	Agriculture & Livestock	17.95%	2.56%	0.91%	0.00%	7.04%	1.00%	7.53%	1.00%	10.00%	1.00%
	Energy		2.56%		0.00%		1.00%		1.00%		1.00%
	Industrial Metals		2.56%		0.00%		1.00%		1.00%		1.00%
	Silver		2.56%		0.00%		1.00%		1.00%		1.00%
	Gold		2.56%		0.90%		1.00%		1.00%		1.00%

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	Palladium		2.56%		0.01%		1.04%		1.53%		4.00%
	Platinum		2.56%		0.00%		1.00%		1.00%		1.00%
Collectables	Art	10.26%	2.56%	0.51%	0.00%	10.44%	1.00%	11.95%	1.00%	10.00%	1.00%
	Wine		2.56%		0.00%		1.00%		1.00%		1.00%
	Whisky		2.56%		0.51%		7.44%		8.95%		7.00%
	Cars		2.56%		0.00%		1.00%		1.00%		1.00%
Cryptocurrencies	General	2.56%	2.56%	0.05%	0.05%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Hedge funds	General	2.56%	2.56%	0.00%	0.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%

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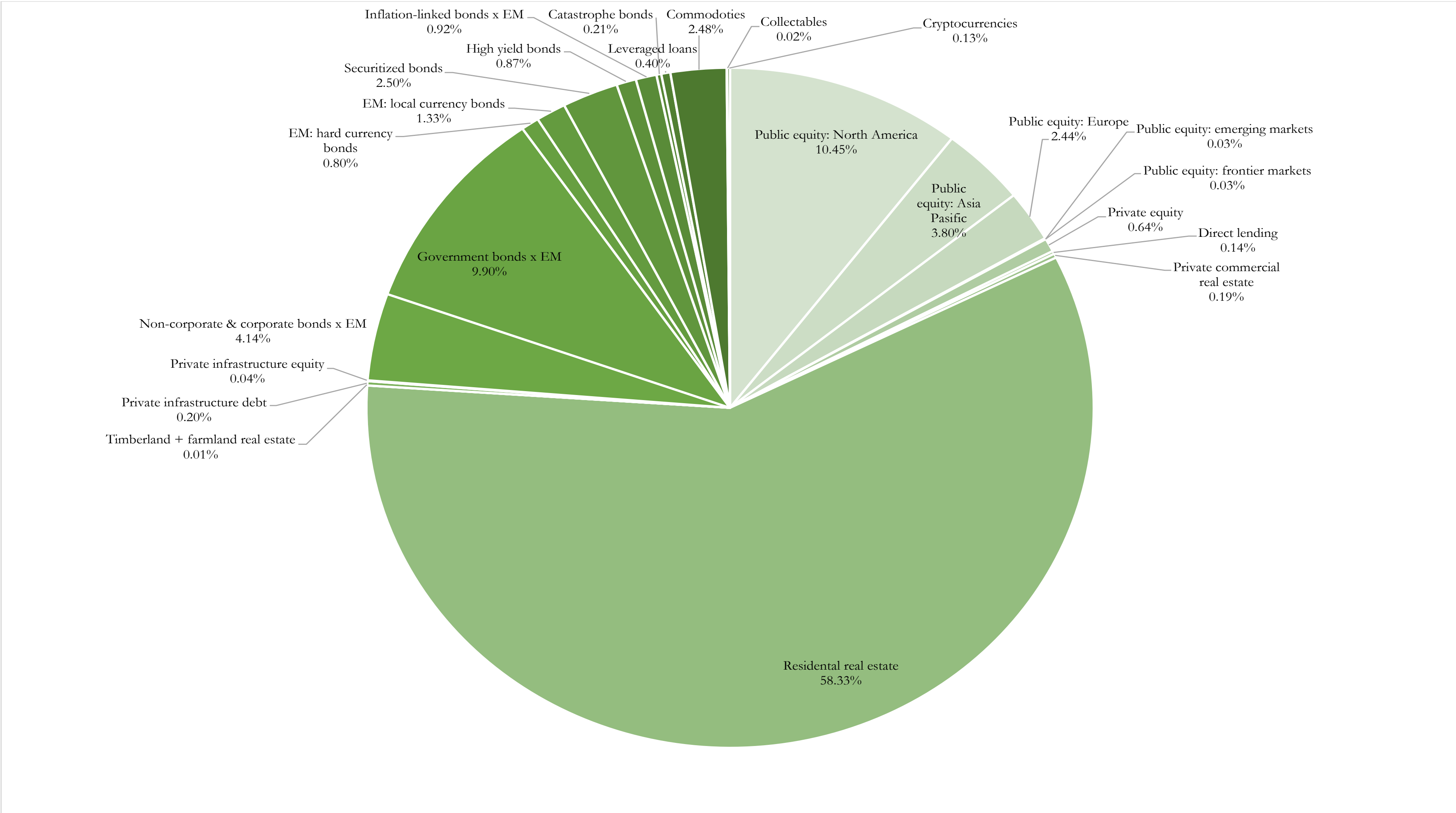

Figure A1. The global market portfolio, divided into further sub-asset classes for demonstrational purpose.

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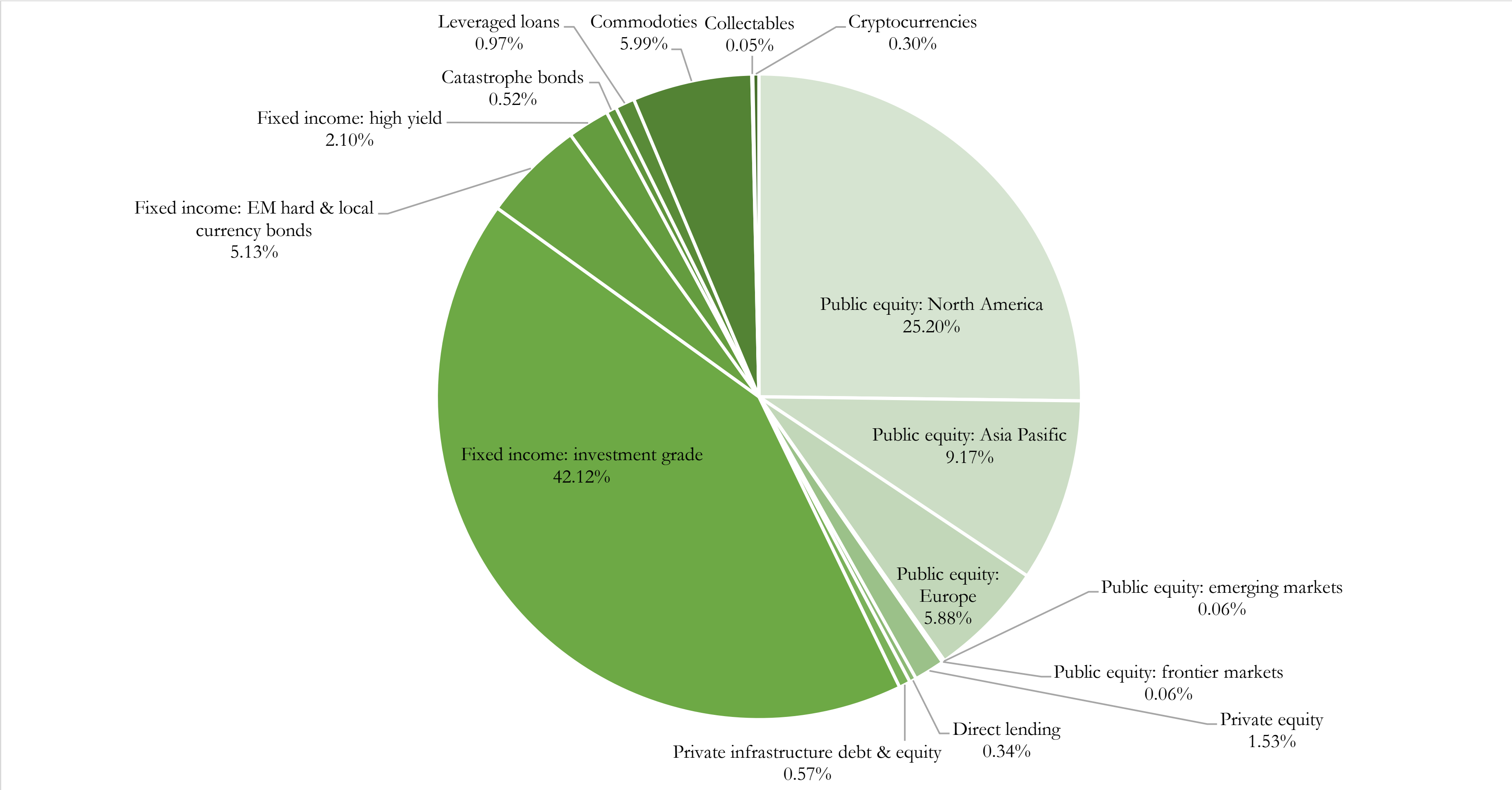

Figure A2. The global market portfolio excluding private real estate. Note that certain sub-asset classes[17] have been merged for illustrative purposes.

[17] Investment grade = non-corporate & corporate bonds ex EM + government bonds ex EM + securitized bonds + inflation-linked bonds. Other assets are categorized according to previously defined asset classes, else defined in Figure A1.

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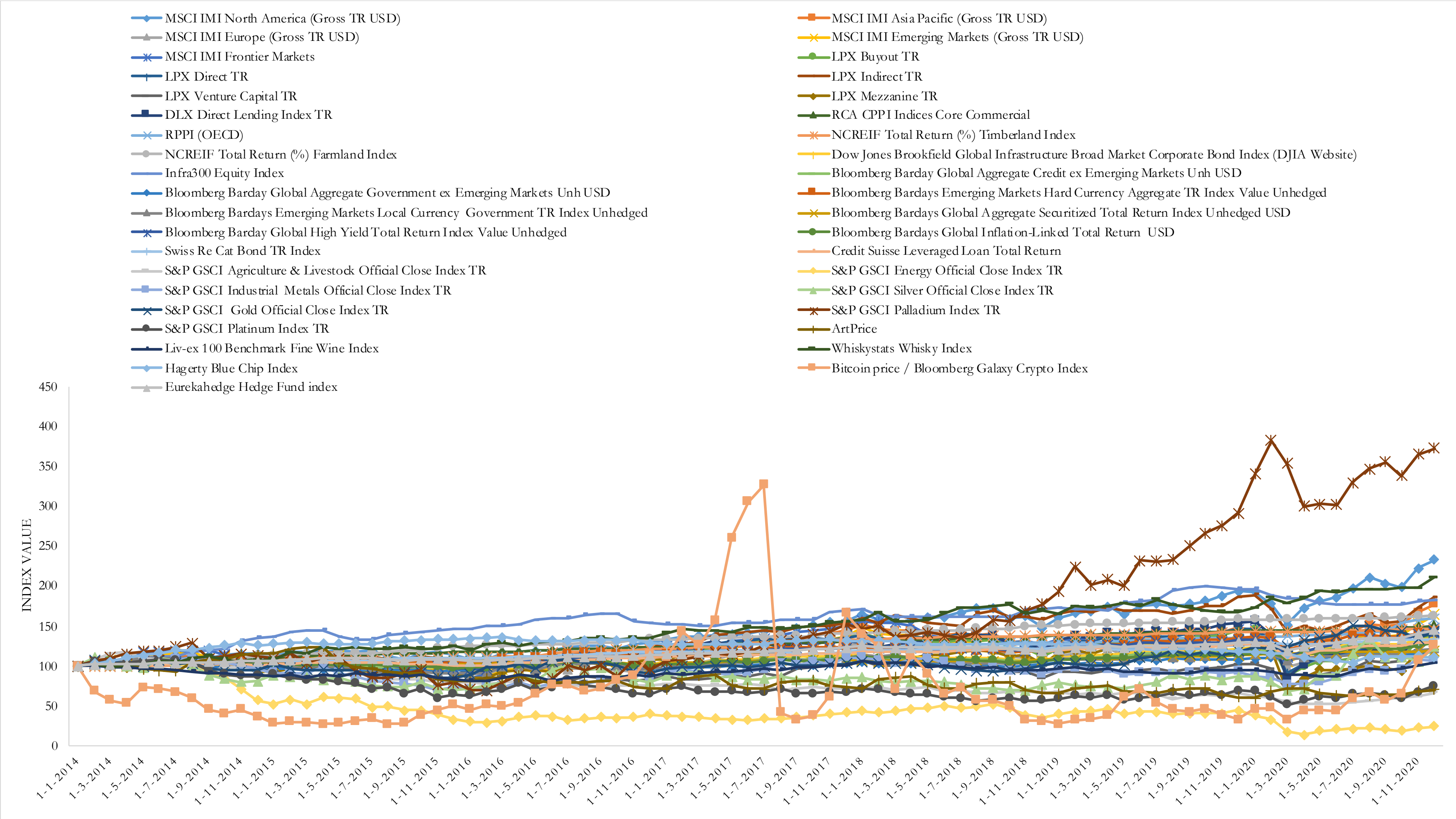

Figure A3. Sub-asset class return data January 2014 (base 100) to December 2020.

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Table A5. Expected return, standard deviation, Sharpe ratio, skewness, kurtosis, LPSD, Sortino ratio, 95% VaR and 95% ES for an equal weighted portfolio and four optimal portfolios, when excluding the sub-asset class private commercial real estate (RCA), 2014-2020.

	P1xRCA	**P2xRCA**	**P3xRCA**	**P4xRCA**	**P5xRCA**
Expected return	6.68%	6.10%	7.22%	7.60%	7.54%
Standard deviation	2.66%	0.34%	1.12%	1.22%	1.40%
Sharpe ratio	2.17	15.20	5.65	5.47	4.72
Skewness	-0.97	1.18	-1.09	-1.01	-0.94
Kurtosis	6.45	3.03	6.85	5.72	5.11
LPSD	2.38%	0.03%	0.46%	0.51%	0.64%
Sortino ratio	2.42	164.93	13.68	13.13	10.39
95% VaR	-1.74%	0.17%	-0.42%	-0.65%	-0.78%
95% ES	-3.46%	0.05%	-1.18%	-1.32%	-1.62%

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Table A6. Optimal weights for each asset class and sub-asset class from the portfolio optimization, excluding private commercial real estate (RCA), 2014-2020.

Asset class	**Sub-Category**	**P1xRCA**		**P2xRCA**		**P3xRCA**		**P4xRCA**		**P5xRCA**	
Public equity	North America	13.16%	2.63%	0.06%	0.06%	5.00%	1.00%	5.00%	1.00%	9.00%	5.00%
	Asia Pacific		2.63%		0.00%		1.00%		1.00%		1.00%
	Europe		2.63%		0.00%		1.00%		1.00%		1.00%
	Emerging		2.63%		0.00%		1.00%		1.00%		1.00%
	Frontier		2.63%		0.00%		1.00%		1.00%		1.00%
Private equity	Buy-out	13.16%	2.63%	0.60%	0.00%	5.00%	1.00%	5.00%	1.00%	5.00%	1.00%
	Direct investment		2.63%		0.00%		1.00%		1.00%		1.00%
	Indirect investment		2.63%		0.60%		1.00%		1.00%		1.00%
	Venture capital		2.63%		0.00%		1.00%		1.00%		1.00%
	Mezzanine		2.63%		0.00%		1.00%		1.00%		1.00%
Direct lending	Direct lending	2.63%	2.63%	0.00%	0.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Private real estate	Residental real estate	7.89%	2.63%	77.14%	19.66%	58.00%	7.73%	43.88%	20.00%	30.00%	19.00%
	Timberland		2.63%		20.79%		1.00%		3.88%		1.00%
	Farmland		2.63%		36.69%		49.27%		20.00%		10.00%
Private infrastructure	Infrastructure debt	5.26%	2.63%	0.00%	0.00%	2.51%	1.00%	12.41%	1.00%	10.00%	1.00%
	Infrastructure equity		2.63%		0.00%		1.51%		11.41%		9.00%

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Fixed income	Global (ex EM) Non-corporate + Corporate	23.68%	2.63%	19.05%	0.00%	9.00%	1.00%	9.00%	1.00%	23.00%	1.00%
	Global (ex EM) Government		2.63%		0.00%		1.00%		1.00%		2.50%
	EM: Hard Currency		2.63%		0.00%		1.00%		1.00%		1.00%
	EM: Local Currency		2.63%		0.00%		1.00%		1.00%		1.00%
	Global: Securitized		2.63%		8.28%		1.00%		1.00%		4.50%
	Global High Yield		2.63%		0.00%		1.00%		1.00%		1.00%
	Global (ex EM) Inflation-linked		2.63%		0.00%		1.00%		1.00%		1.00%
	CAT Bond		2.63%		10.78%		1.00%		1.00%		10.00%
	Leveraged Loans		2.63%		0.00%		1.00%		1.00%		1.00%
Commodoties	Agriculture & Livestock	18.42%	2.63%	0.73%	0.00%	7.05%	1.00%	7.95%	1.00%	10.00%	1.00%
	Energy		2.63%		0.00%		1.00%		1.00%		1.00%
	Industrial Metals		2.63%		0.00%		1.00%		1.00%		1.00%
	Silver		2.63%		0.00%		1.00%		1.00%		1.00%
	Gold		2.63%		0.67%		1.00%		1.00%		1.00%
	Palladium		2.63%		0.05%		1.05%		1.95%		4.00%
	Platinum		2.63%		0.00%		1.00%		1.00%		1.00%

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Collectables	Art	10.53%	2.63%	2.43%	0.45%	10.45%	1.00%	13.76%	1.00%	10.00%	1.00%
	Wine		2.63%		0.00%		1.00%		1.00%		1.00%
	Whisky		2.63%		1.98%		7.45%		10.76%		7.00%
	Cars		2.63%		0.00%		1.00%		1.00%		1.00%
Cryptocurrencies	General	2.63%	2.63%	0.00%	0.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Hedge funds	General	2.63%	2.63%	0.00%	0.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%

Table A7. Expected return, standard deviation, Sharpe ratio, skewness, kurtosis, LPSD, Sortino ratio, 95% VaR and 95% ES for an equal weighted portfolio and four optimal portfolios, when excluding the private real estate asset class, 2014-2020.

	P1xRE	**P2xRE**	**P3xRE**	**P4xRE**	**P5xRE**
Expected return	6.70%	6.53%	9.52%	8.24%	9.20%
Standard deviation	2.88%	0.81%	1.75%	1.56%	2.24%
Sharpe ratio	2.00	6.91	4.91	4.69	3.69
Skewness	-0.96	-0.75	-0.77	-1.01	-0.53
Kurtosis	6.39	1.50	2.69	3.83	2.25
LPSD	2.78%	0.19%	0.77%	0.72%	1.17%
Sortino ratio	2.08	29.89	11.14	10.13	7.09
95% VaR	-1.93%	-0.31%	-0.90%	-1.01%	-1.52%
95% ES	-3.80%	-0.93%	-2.27%	-2.04%	-2.96%

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Table A8. Optimal weights for each asset class and sub-asset class from the portfolio optimization, excluding the asset class private real estate, 2014-2020.

Asset class	Sub-Category	P1xRE		P2xRE		P3xRE		P4xRE		P5xRE	
Public equity	North America	14.29%	2.86%	2.30%	2.30%	5.00%	1.00%	5.00%	1.00%	17.39%	10.39%
	Asia Pacific		2.86%		0.00%		1.00%		1.00%		2.50%
	Europe		2.86%		0.00%		1.00%		1.00%		2.50%
	Emerging		2.86%		0.00%		1.00%		1.00%		1.00%
	Frontier		2.86%		0.00%		1.00%		1.00%		1.00%
Private equity	Buy-out	14.29%	2.86%	2.51%	0.00%	5.00%	1.00%	5.00%	1.00%	5.00%	1.00%
	Direct investment		2.86%		0.00%		1.00%		1.00%		1.00%
	Indirect investment		2.86%		2.51%		1.00%		1.00%		1.00%
	Venture capital		2.86%		0.00%		1.00%		1.00%		1.00%
	Mezzanine		2.86%		0.00%		1.00%		1.00%		1.00%
Direct lending	Direct lending	2.86%	2.86%	0.00%	0.00%	1.00%	1.00%	1.00%	1.00%	1.00%	1.00%
Private infrastructure	Infrastructure debt	5.71%	2.86%	16.95%	0.00%	35.78%	1.00%	21.00%	1.00%	10.00%	1.00%
	Infrastructure equity		2.86%		16.95%		34.78%		20.00%		9.00%
Fixed income	Global (ex EM) Non-corporate + Corporate	25.71%	2.86%	61.49%	0.00%	9.00%	1.00%	28.00%	1.00%	30.00%	2.50%
	Global (ex EM) Government		2.86%		0.00%		1.00%		1.00%		5.00%

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	EM: Hard Currency		2.86%		0.00%		1.00%		1.00%		1.00%
	EM: Local Currency		2.86%		0.00%		1.00%		1.00%		1.00%
	Global: Securitized		2.86%		23.66%		1.00%		1.00%		7.50%
	Global High Yield		2.86%		0.00%		1.00%		1.00%		1.00%
	Global (ex EM) Inflation-linked		2.86%		0.00%		1.00%		1.00%		1.00%
	CAT Bond		2.86%		37.82%		1.00%		20.00%		10.00%
	Leveraged Loans		2.86%		0.00%		1.00%		1.00%		1.00%
Commodoties	Agriculture & Livestock	20.00%	2.86%	2.11%	0.00%	12.04%	1.00%	12.10%	1.00%	20.00%	1.00%
	Energy		2.86%		0.00%		1.00%		1.00%		1.00%
	Industrial Metals		2.86%		0.00%		1.00%		1.00%		1.00%
	Silver		2.86%		0.00%		1.00%		1.00%		1.00%
	Gold		2.86%		0.00%		1.00%		1.00%		5.00%
	Palladium		2.86%		2.11%		6.04%		6.10%		10.00%
	Platinum		2.86%		0.00%		1.00%		1.00%		1.00%
Collectables	Art	11.43%	2.86%	14.47%	0.00%	30.18%	1.00%	25.90%	1.00%	10.00%	1.00%
	Wine		2.86%		0.00%		1.00%		1.00%		1.00%
	Whisky		2.86%		14.47%		27.18%		20.00%		7.00%
	Cars		2.86%		0.00%		1.00%		3.90%		1.00%
Cryptocurrencies	General	2.86%	2.86%	0.16%	0.16%	1.00%	1.00%	1.00%	1.00%	2.24%	2.24%

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Hedge funds	General	2.86%	2.86%	0.00%	0.00%	1.00%	1.00%	1.00%	1.00%	4.38%	4.38%

Table A9. Optimal weights for each asset class and sub-asset class from the portfolio optimization, based on 30 indices 2003-2020.

Asset class	**Sub-Category**	**P1***		**P2***		**P3***		**P4***		**P5***	
Public equity	North America	13.33%	3.33%	0.01%	0.00%	4.00%	1.00%	10.00%	5.00%	8.00%	5.00%
	Asia Pacific		3.33%		0.01%		1.00%		3.00%		1.00%
	Europe		3.33%		0.00%		1.00%		1.00%		1.00%
	Emerging		3.33%		0.00%		1.00%		1.00%		1.00%
Private equity	Buy-out	16.67%	3.33%	0.78%	0.00%	5.15%	1.00%	5.00%	1.00%	5.00%	1.00%
	Direct investment		3.33%		0.00%		1.00%		1.00%		1.00%
	Indirect investment		3.33%		0.78%		1.15%		1.00%		1.00%
	Venture capital		3.33%		0.00%		1.00%		1.00%		1.00%
	Mezzanine		3.33%		0.00%		1.00%		1.00%		1.00%
Private Real Estate	Private commercial real estate	13.33%	3.33%	33.07%	5.90%	27.85%	4.93%	20.00%	5.00%	30.00%	1.00%
	Residental real estate		3.33%		7.40%		6.21%		5.00%		9.00%
	Timberland		3.33%		8.25%		6.90%		5.00%		10.00%
	Farmland		3.33%		11.53%		9.80%		5.00%		10.00%
Private Infrastructure	Infrastructure equity	3.33%	3.33%	11.07%	11.07%	9.57%	9.57%	5.00%	5.00%	10.00%	10.00%

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Fixed income	Global Non-corporate + Corporate	23.33%	3.33%	35.65%	4.47%	30.95%	3.93%	35.00%	5.00%	29.00%	1.00%
	Global Government		3.33%		4.68%		4.08%		5.00%		5.00%
	Global: Securitized		3.33%		5.68%		4.83%		5.00%		10.00%
	Global High Yield		3.33%		4.23%		3.83%		5.00%		1.00%
	Global (ex EM) Inflation-linked		3.33%		4.30%		3.82%		5.00%		1.00%
	CAT Bond		3.33%		7.75%		6.54%		5.00%		10.00%
	Leveraged Loans		3.33%		4.55%		3.91%		5.00%		1.00%
Commodoties	Agriculture & Livestock	20.00%	3.33%	6.37%	0.00%	10.82%	1.00%	10.00%	1.00%	6.00%	1.00%
	Energy		3.33%		0.00%		1.00%		1.00%		1.00%
	Industrial Metals		3.33%		0.06%		1.00%		1.00%		1.00%
	Silver		3.33%		0.82%		1.68%		1.00%		1.00%
	Gold		3.33%		5.49%		5.14%		5.00%		1.00%
	Platinum		3.33%		0.00%		1.00%		1.00%		1.00%
Collectables	Art	6.67%	3.33%	7.05%	3.58%	6.48%	3.29%	10.00%	5.00%	2.00%	1.00%
	Wine		3.33%		3.47%		3.19%		5.00%		1.00%
Hedge funds	General	3.33%	3.33%	5.99%	5.99%	5.18%	5.18%	5.00%	5.00%	10.00%	10.00%

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Table A9. Optimal weights for 60/40 stock-bond portfolios, based on eight indices, 2014-2020.

Asset class	Sub-Category	M1		M2		M3		M4		M5	
Public equity	North America	60.00%	12.00%	60.00%	60.00%	60.00%	56.00%	60.00%	20.00%	60.00%	37.46%
	Asia Pacific		12.00%		0.00%		1.00%		20.00%		13.62%
	Europe		12.00%		0.00%		1.00%		1.00%		8.73%
	Emerging		12.00%		0.00%		1.00%		1.00%		0.09%
	Frontier		12.00%		0.00%		1.00%		18.00%		0.10%
Fixed income	Global Non-corporate + Corporate x EM	40.00%	13.33%	40.00%	0.00%	40.00%	1.00%	40.00%	20.00%	40.00%	11.06%
	Global Government x EM		13.33%		40.00%		38.00%		17.46%		26.46%
	Inflation Linked		13.33%		0.00%		1.00%		2.54%		2.47%

Table A10. Optimal weights for 60/40 stock-bond portfolios, based on seven indices, 2003-2020.

Asset class	Sub-Category	M1*		M2*		M3*		M4*		M5*	
Public equity	North America	60.00%	15.00%	60.00%	60.00%	60.00%	57.00%	60.00%	20.00%	60.00%	37.52%
	Asia Pacific		15.00%		0.00%		1.00%		20.00%		13.65%
	Europe		15.00%		0.00%		1.00%		1.00%		8.75%
	Emerging		15.00%		0.00%		1.00%		19.00%		0.09%
Fixed income	Global Non-corporate + Corporate	40.00%	13.33%	40.00%	0.00%	40.00%	1.00%	40.00%	2.24%	40.00%	11.06%
	Global Government		13.33%		40.00%		38.00%		20.00%		26.46%
	Inflation Linked		13.33%		0.00%		1.00%		17.76%		2.47%

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Table A11. Two factor ANOVA with replication on alphas.

Source of Variation	SS	df	MS	F	P-value	F crit
Sample	1.20E-04	1	1.20E-04	257.26	1.83E-08	4.96
Columns	6.43E-05	4	1.61E-05	34.57	7.90E-06	3.48
Interaction	2.10E-05	4	5.26E-06	11.31	9.90E-04	3.48
Within	4.65E-06	10	4.65E-07			
Total	2.09E-04	19				

Notes: The ANOVA is carried out on two samples (optimal risky portfolios and 60/40 balanced portfolios), five columns (the five portfolios) and two factors (the two time periods 2014-2020 and 2003-2020).

Table A12. Two factor ANOVA with replication on Sharpe ratios.

Source of Variation	SS	df	MS	F	P-value	F crit
Sample	79.36	1	79.36	4.88	0.05	4.96
Columns	114.99	4	28.75	1.77	0.21	3.48
Interaction	98.07	4	24.52	1.51	0.27	3.48
Within	162.58	10	16.26			
Total	454.99	19				

Notes: The ANOVA is carried out on two samples (optimal risky portfolios and 60/40 balanced portfolios), five columns (the five portfolios) and two factors (the two time periods 2014-2020 and 2003-2020).

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Table A13. Two factor ANOVA with replication on Sortino ratios.

Source of Variation	**SS**	**df**	**MS**	**F**	**P-value**	**F crit**
Sample	4483.76	1	4483.76	1.73	0.22	4.96
Columns	12131.83	4	3032.96	1.17	0.38	3.48
Interaction	11681.31	4	2920.33	1.13	0.40	3.48
Within	25946.72	10	2594.67			
Total	54243.62	19				

Notes: The ANOVA is carried out on two samples (optimal risky portfolios and 60/40 balanced portfolios), five columns (the five portfolios) and two factors (the two time periods 2014-2020 and 2003-2020).

Table A14. Descriptive statistics based on robustness test (resample with replacement) for portfolio P1-P4 covering the period 2014-2020.

	(P1)				**(P2)**				**(P3)**				**(P4)**			
	Min	Max	Average	Median	Min	Max	Average	Median	Min	Max	Average	Median	Min	Max	Average	Median
E(r)	-4.77%	12.58%	5.37%	5.49%	5.21%	6.43%	5.89%	5.89%	3.17%	10.22%	6.77%	6.76%	2.72%	10.30%	6.85%	6.87%
St.Dev	1.57%	3.61%	2.30%	2.23%	0.14%	0.24%	0.20%	0.20%	0.68%	1.44%	0.92%	0.89%	0.71%	1.54%	0.97%	0.93%
Sharpe	-1.71	6.13	2.09	1.96	20.59	33.43	24.78	24.54	1.67	11.27	6.64	6.55	1.28	11.75	6.41	6.27
LPSD	0.32%	4.36%	1.61%	1.88%	0.01%	0.03%	0.02%	0.02%	0.08%	0.73%	0.28%	0.32%	0.08%	0.81%	0.31%	0.36%
Sortino	-2.11	30.59	5.70	2.97	171.18	358.87	254.99	259.48	3.32	91.17	33.75	20.55	2.40	95.79	32.31	19.41

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Table A15. Descriptive statistics based on robustness test (resample with replacement) for portfolio P1*-P4* covering the period 2003-2020.

	(P1*)				**(P2*)**				**(P3*)**				**(P4*)**			
	Min	Max	Average	Median	Min	Max	Average	Median	Min	Max	Average	Median	Min	Max	Average	Median
E(r)	0.98%	12.38%	6.64%	6.73%	5.29%	9.23%	7.36%	7.43%	4.65%	10.23%	7.28%	7.41%	3.68%	10.00%	6.72%	6.87%
St.Dev	2.09%	3.58%	2.89%	2.89%	0.81%	1.26%	1.01%	1.01%	1.14%	1.87%	1.47%	1.48%	1.30%	2.21%	1.75%	1.76%
Sharpe	0.02	5.48	2.05	1.99	4.21	9.22	6.47	6.35	2.35	7.70	4.41	4.33	1.48	6.91	3.40	3.37
LPSD	0.64%	5.04%	2.83%	2.90%	0.11%	0.70%	0.35%	0.38%	0.21%	1.48%	0.78%	0.82%	0.30%	2.17%	1.14%	1.20%
Sortino	0.02	17.81	2.67	1.95	7.89	70.28	23.55	17.25	3.17	42.17	10.44	7.94	1.61	30.27	6.69	5.03

Table A16. The 95 percent confidence interval for VaR, for portfolios P1 to P5 (2014-2020) and L1 to L5 (2003-2020), based on resample with replacement.

	(P1)	**(P2)**	**(P3)**	**(P4)**	**(P5)**
2.50%	-1.63%	0.31%	-0.32%	-0.40%	-0.76%
97.50%	-10.95%	-0.04%	-3.96%	-4.21%	-5.37%
	(P1*)	**(P2*)**	**(P3*)**	**(P4*)**	**(P5*)**
2.50%	-3.83%	-1.06%	-1.43%	-2.00%	-1.08%
97.50%	-11.64%	-2.62%	-5.20%	-6.25%	-5.05%

Notes: The lower (2.5%) and upper (97.5%) bound of the 95% confidence interval for Va

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